Nonlinear Vibrations of Long Slender Continua Coupled with Discrete Inertia Elements Moving Vertically in a Tall Structure

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Summary. A nonlinear mathematical model of axially moving long slender continua coupled with discrete inertia elements deployed in a tall structure is developed. In engineering applications such a system represents a vertical transportation system (high-rise lift/elevator). The longitudinal motion of the discrete masses that represent the car, counterweight and compensating sheave assembly are constrained by a a nonlinear damping device. Numerical simulation techniques are then used to predict a range of complex dynamic interactions and resonance phenomena which in turn informs the development of vibration control strategies.

Introduction

Tall buildings and structures are subjected to sway motions of large amplitude and low frequency due to resonance conditions induced by wind loads and long-period seismic excitations [1,2]. These sources of excitation affect the performance of vertical transportation systems (VTS; high-rise lift/elevator systems) deployed in buildings [3]. The fundamental natural frequencies of tall buildings fall within the frequency range of the wind and seismic excitations and the sway motions form a base motion excitation mechanism which acts upon the components of VTS [4]. Particularly affected are long slender continua (LSC) such as suspension ropes, compensating and travelling cables. The lengths of these elements vary when the VTS moves vertically within the host structure. The lateral motions of the LSC are coupled with the vertical motions of discrete masess installed in the hoisway, such as the car, counterweight of the compensating sheave assembly (CSA).
Complex resonance interactions arise in the system when the frequency of the base excitation is tuned to one (or more) natural frequencies of the system.Substantial research efforts has been devoted to the issue of mitigating the effects of the dynamic responses that might occur. In the first instance the masses and geometry of the system can be adjusted to change the resonance frequencies to shift the resonance regions. However, in most cases the structural constraints and design limitations do not leave much space for the changes to be effective. Active vibration control strateges involving boundary lateral motions and/or longitudinal motions (such as active stiffness control [4]) can be considered to mitigate the effects of resonances. However, these approaches involve the application of expensive and sophisticated actuator control algorithms and often passive methods are preferred to be used. For example, the industrial practice to mitigate the effects of dynamic interactions in a high-rise elevator system involves the application of a hydraulic tie-down device attached at the CSA. The damping force is then a nonlinear function of the CSA's velocity. The aim of this study is to develop a numerical simulation model to predict and to analyse the nonlinear vibrational interactions in the system under resonance conditions. The characteristics of the damping device can then be optimised and adjusted to minimize the effects of adverse dynamic responses of the system.

Nonlinear dynamics model

Figure 1 shows a VTS system mounted within a vertical cantilever host structure subject to ground motions $s_r(t)$, $r = 1,2$ in the in-plane and out-of-plane directions, respectively. The structure undergoes bending elastic deformations with the in-plane and out-of-plane displacements at the top end ($z = z_0$) denoted as $w_{r0}(t)$, respectively. The fundamental modal responses of the structure can be defined by the following equation

$$\ddot{W}_r(t) + 2\zeta_r\omega_r\dot{W}_r(t) + \omega_r^2 W_r(t) = -\frac{\ddot{s}_r(t)}{m_r}\int_0^{Z_0} m_s(z)\,\Psi_r(z)\,dz \tag{1}$$

where $r = 1,2$ and W_r represent the modal coordinates. The natural frequencies of the structure are denoted as ω_r, ζ_r represent the modal damping ratios, $\Psi_r(z)$ are the eigenfunctions (mode shape functions) of the structure. In this formulation $0 \le z \le z_0$, $m_s(z)$ is the linear mass density of the structure and $m_r = \int_0^{z_0} m_s(z)\Psi_r^2(z)dz$. The deflections at the top of the structure are then be determined as $w_{r0} = \Psi_r(Z_0)W_r(t)$, $r = 1,2$. The equations of motion of the LSC treated as continua with small bending stiffness are given as

$$\begin{aligned}
&m_i\bar{v}_{itt} + E_iJ_i\bar{v}_{ixxxx} - \left\{T_i - m_i\left[V^2 + (g - a_i)x_i\right] + E_iA_ie_i\right\}\bar{v}_{ixx} + m_ig\bar{v}_{ix} + 2m_iV\bar{v}_{ixt} = F_i^v\left[t, L_i(t)\right],\ i = 1,\dots 4,\\
&m_i\bar{w}_{itt} + E_iJ_i\bar{w}_{ixxxx} - \left\{T_i - m_i\left[V^2 + (g - a_i)x_i\right] + E_iA_ie_i\right\}\bar{w}_{ixx} + m_ig\bar{w}_{ix} + 2m_iV\bar{w}_{ixt} = F_i^w\left[t, L_i(t)\right],\ i = 1,\dots 4.
\end{aligned} \tag{2}$$

The response of discrete masses are desribed by following equations

$$\begin{aligned}
&M_1\ddot{q}_1 - E_1A_1e_1 + E_2A_2e_3 = 0;\ M_2\ddot{q}_2 - E_1A_1e_4 + E_2A_2e_2 = 0,\\
&M_3\ddot{q}_3 + E_1A_1e_1 + E_1A_1e_4 + c_3\left|\dot{q}_3\right|^{\alpha-1}\dot{q}_3 = 0;\ I_3\ddot{\theta}_3 - RE_1A_1e_1 + RE_1A_1e_4 = 0
\end{aligned} \tag{3}$$

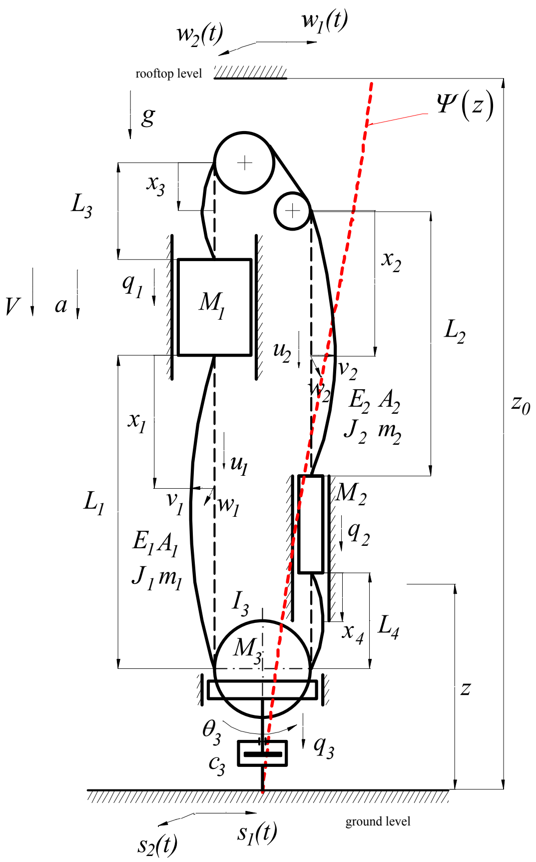

Figure 1: Greatest figure of all time

where $\bar{v}_i(x_i,t), \bar{w}_i(x_i,t)$, i = 1,2,..., 4, represent the dynamic displacements of the LSC, F_i^v, F_i^w are the excitation terms, E_1, A_1, J_1, m_1 and E_2, A_2, J_2, m_2 are the modulus elasticity, cross-sectional effective area, second moment of area, and mass per unit length of the compensating ropes and the suspension ropes, respectively. The compensating ropes are of length L_1 at the car side and the suspension ropes are of length L_2 at the counterweight side, respectively. The length of the suspension rope at the car side and the compensating rope at the counterweight side are denoted as L_3 and L_4, respectively. The lengths of suspension ropes and compensating cables are time-varying so that $L_i = L_i(t)$, $i = 1,\ldots,4$. The masses and dynamic displacements of the car, counterweight and the compensating sheave assembly are represented by M_1, M_2 and M_3, q_1, q_2 and q_3, respectively. The speed and acceleration/ deceleration of the car are denoted by V and a, respectively, and T_i, denote the rope quasi-static tensions. In this model the quantities e_i are the quasi-static axial strains in the LSC system. The kinematic constraint equation to be used in equations (2-3) is expressed as $\Upsilon \equiv 2q_3 - u_1(L_1,t) - u_4(L_4,t) = 0$.

Results and conclusions

The model is solved numerically and the results demonstrate the resonance behaviour of the system. The resonance frequencies of the LSC componets can be shifted / changed by the use of different masses of the CSA. The frequencies of the suspension ropes depend on the mass of the car (and the corresponding mass of the counterweight) as well as on the car loading conditions. The characteristics of the hydraulic tie-down / damping device can be optimised and adjusted to minimize the effects of adverse dynamic interactions taking place in the system. It should be noted that the nature of the dynamic conditions present in high-rise building systems is such that small changes of the natural frequencies of the structure might result in large changes of the resonance conditions that arise in the installation.

References

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