DEA addresses this issue by focusing on the performance of all the factors (Charles & Diaz, 2016). Both models are useful and serve different purposes and hence, our aim is to provide the readers with a comparison between the two.

3.2.1 Radial Pure DEA

Having obtained the factor-level index for every factor through the two-stage OGI, in line with Lovell & Pastor (1999), the following system would produce the DEA-based index of social progress for region o, that has a vector of $|I|$ outputs (the factors) $G_o = (G_o^1, ..., G_o^{|I|})$ and belongs to a set of R regions:

$$\begin{aligned} \max_{\phi,\lambda_1,...,\lambda_R} \quad & \phi \\ \text{s.t.} \quad & \phi G_0^i \leq \sum_{r \in R} G_r^i \lambda_r, \quad \forall i \in I \\ & \sum_{r \in R} \lambda_r = 1, \quad \lambda_r \geq 0, \quad \forall r \in R. \end{aligned} \tag{6}$$

The result of this system would produce a vector $(\phi_o, \lambda_{o1}, ..., \lambda_{oR})$ for region o. Upon solving System (6) for every region, $r \in R$, we use the results to construct a social progress index $\theta_r^{radial} = 1/\phi_r$ for each of the regions.

3.2.2 Non-radial Pure DEA

Let us consider System (6) in light of the non-radial DEA (Charles & Zegarra, 2014), wherein importance has been given to all the factors under study. Similarly to the radial approach, we use the factor-level index obtained from the two-stage OGI and plug it into the following System (7). This in turn produces the non-radial pure DEA-based index of social progress for region o, which has a vector of $|I|$ outputs (the factors) $G_o = (G_o^1, ..., G_o^{|I|})$ and belongs to a set of R regions:

$$\begin{aligned} \max_{\Phi^1,...,\Phi^{|I|},\lambda_1,...,\lambda_R} \quad & \sum_{i \in I} \Phi^i \\ \text{s.t.} \quad & \Phi^i G_o^i \leq \sum_{r \in R} G_r^i \lambda_r, \quad \forall i \in I \\ & \sum_{r \in R} \lambda_r = 1, \quad \lambda_r \geq 0, \quad \forall r \in R. \end{aligned} \tag{7}$$

The result of this system would produce a vector $(\Phi_o^1, ..., \Phi_o^{|I|}, \lambda_{o1}, ..., \lambda_{oR})$ for region o. Upon solving System (7) for every region, $r \in R$, we use the results to construct a social progress index $\theta_r^{non-radial} = |I| / \sum_{i \in I} \Phi_r^i$ for each of the regions.

4. Inferences from the Analytics

Table 1 shows the OGI indices for all the three factors of social progress, the social progress indices for the year 2015, as well as the associated ranking, for both the radial and non-radial

DEA applied in the second phase of our modelling. It is to be noted that radial DEA refers to the indices obtained by running the pure DEA method with radial expansion (System 6), while the non-radial DEA refers to the indices attained by running the non-radial pure DEA method (System 7).

The interpretation of the results is quite straightforward. First, the table shows the situation of the regions in view of each of the three factors considered, that is Basic Human Needs, Foundations of Wellbeing, and Opportunity. We can quickly observe that the best performers in the BHN factor are Ica, Callao, and Lambayeque, while the worst performers are Madre de Dios, followed by Puno. Similarly, Tacna, Cusco, Huancavelica, and Ancash are the best performers in the FoW factor, with Ucayali, Lima Provincias, and Loreto as the worst performers. Lastly, in the Opp factor, Moquequa, Ica, Lambayeque, Ancash, and La Libertad are the best performers and Puno, Loreto, Madre de Dios, Pasco, Amazonas, Ucayali, and Huanuco are the worst performers.

Second, the table provides the ranking of the regions based on both the radial and non-radial DEA-based SPI. An immediate observation is that, independent of method used and despite some variations, the coastal regions are in general the highest performers in terms of social progress, while the jungle regions tend to be the worst performers. This finding is consistent with previous research (Charles & D'Alessio, 2019). A visual representation of the results will yield additional insights and this is what we proceed to do in the next sub-section.

Table 1
SPI: Ranking of Regions based on Radial and Non-Radial Pure DEA

DMU	Regions	BHN	FoW	Opp	SPI-R	Rank-R	SPI-NR	Rank-NR
1	Amazonas[3]	18.51	20.77	9.63	0.9168	16	0.8033	19
2	Ancash[1]	18.31	22.02	14.18	0.9956	2	0.9377	7
3	Apurimac[2]	17.59	20.88	12.23	0.9219	15	0.8655	13
4	Arequipa[1]	20.23	21.65	13.57	0.9793	4	0.9526	4
5	Ayacucho[2]	19.93	21.94	12.24	0.9761	7	0.9174	10
6	Cajamarca[2]	17.08	21.64	12.12	0.9453	12	0.8625	15
7	Callao[1]	21.51	19.59	11.34	0.9858	3	0.8818	12
8	Cusco[2]	17.07	22.52	10.08	0.9674	10	0.8139	18
9	Huancavelica[2]	16.71	22.17	12.12	0.9630	11	0.8613	16
10	Huanuco[2]	17.40	21.13	9.76	0.9076	18	0.7953	20
11	Ica[1]	21.82	21.38	14.70	1.0000	1	1.0000	1
12	Junin[2]	17.21	21.32	12.22	0.9364	13	0.8637	14
13	La Libertad[1]	18.05	21.56	14.01	0.9771	6	0.9231	8
14	Lambayeque[1]	21.36	19.88	14.19	0.9789	5	0.9576	3
15	Lima Metropolitana[1]	20.01	21.57	13.25	0.9709	8	0.9401	6
16	Lima Provincias[1]	17.46	16.84	10.54	0.8002	21	0.7665	21
17	Loreto[3]	17.37	16.45	8.51	0.7961	22	0.7004	24
18	Madre de Dios[3]	13.49	17.71	9.51	0.7662	23	0.6864	26
19	Moquegua[1]	20.23	21.76	14.90	1.0000	1	0.9843	2
20	Pasco[2]	16.92	19.33	9.51	0.8470	20	0.7612	22
21	Piura[1]	20.32	21.30	13.40	0.9677	9	0.9450	5
22	Puno[2]	15.49	21.18	7.61	0.9098	17	0.6897	25
23	San Martin[3]	18.80	19.80	11.70	0.8975	19	0.8579	17
24	Tacna[1]	19.39	23.28	12.11	1.0000	1	0.9209	9
25	Tumbes[1]	20.36	18.98	12.28	0.9331	14	0.8836	11
26	Ucayali[3]	16.56	15.16	9.68	0.7589	24	0.7065	23
Statistics	Min	13.49	15.16	7.61	0.7589	1	0.6864	1
	Q1	17.11	19.64	9.84	0.9082	5.25	0.7973	7.25
	Median	18.05	21.18	12.12	0.9453	11	0.8637	13
	Q3	20.18	21.65	13.36	0.9785	17.75	0.9341	19.75
	Max	21.82	23.28	14.9	1.0000	24	1.0000	26
	Mean	18.43	20.45	11.75	0.9269	11.62	0.8569	13.5
	SD	2.00	2.00	2.00	0.0700	7.47	0.0900	7.65

Note: BHN - Basic Human Needs, FoW - Foundations of Wellbeing, Opp - Opportunity, SPI - Social Progress Index. $^{1} = Coast$,$^{2} = Highlands$,$^{3} = Jungle$.

4.1. Visual analytics

Figure 2 presents the boxplots of factors vs. classifications. With three factors and three classifications, the figure shows a total of nine boxplots, wherein each boxplot represents a specific factor and a specific classification. Furthermore, there are three median lines drawn horizontally in red colour; these represent the respective factor medians and are provided along with their 95% median confidence intervals. Among the three factor medians, the lowest median is recorded in the case of the Opp factor, followed by the BHN factor and then the FoW factor.

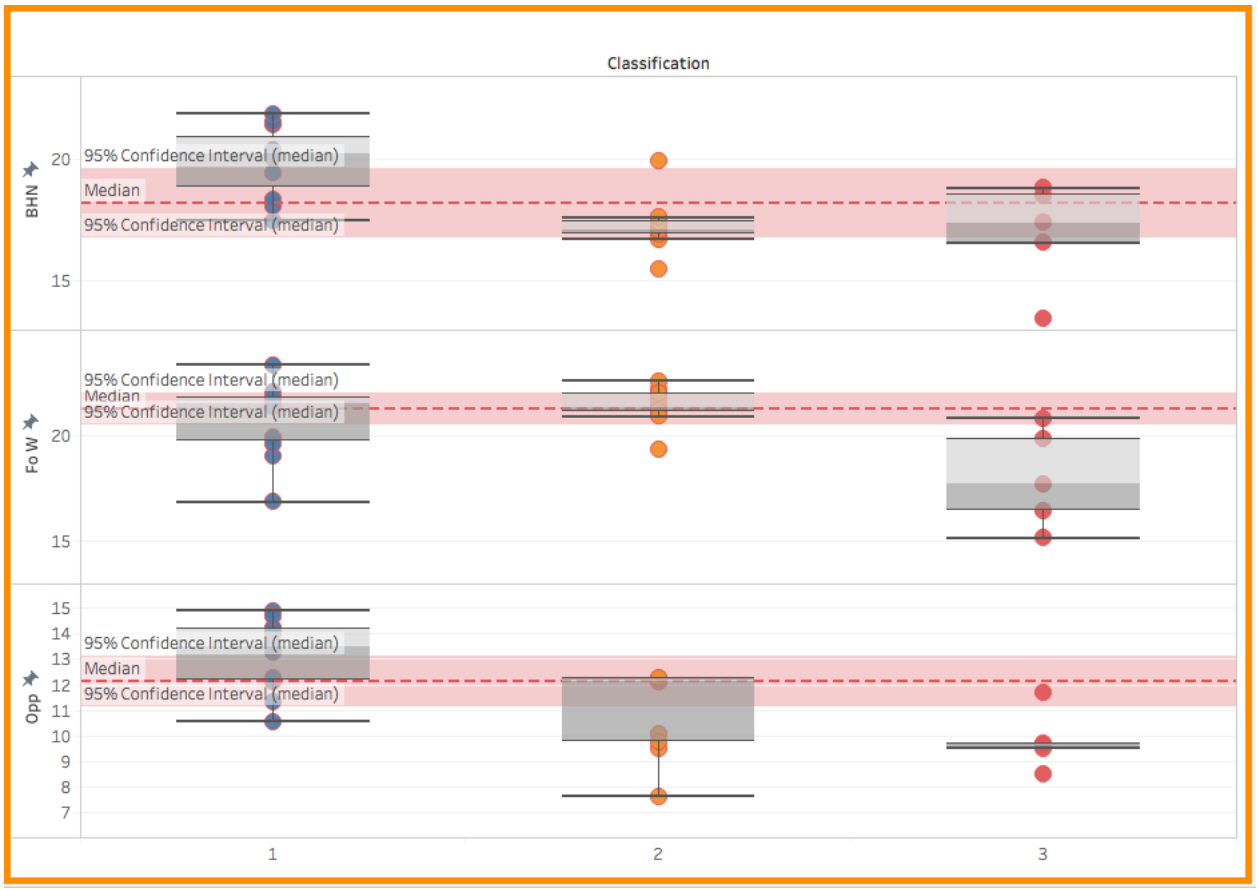

Fig. 2. Boxplots of Factors vs. Classifications.

[a]Note. 1 = Coastal regions; 2 = Highlands regions; 3 = Jungle regions.

Although based on Table 1, one can observe that the standard deviations of the three factors are the same, still, in Figure 2, we can see that there is a difference in the width of the confidence interval. The variation is high in the case of BHN when compared to the other two factors, FoW and Opp; by contrast, FoW registers a narrower confidence interval. We can observe that in the case of the BHN factor, the median of the coastal region falls outside the interval, while the other two medians of the highlands and jungle regions fall inside the interval. In the case of the FoW factor, the first two medians of the coastal and highlands regions fall inside the interval, while the third median of the jungle region falls outside the interval. Lastly, in the case of the Opp factor, the medians of all the regions fall outside the interval. Overall, the interesting observation to make here is that in two out of three cases

(for FoW and Opp), the medians of the third classification (that is, the jungle) are below the respective factor medians. This means that the jungle regions perform particularly poorly in the FoW and Opp factors. By contrast, in two out of three cases (for BHN and Opp), the medians of the first classification (that is, the coast) are above the respective factor medians. In other words, the coastal regions are performing particularly well in the BHN and Opp factors.

Figure 3 shows the correlations among the three factors, BHN, FoW, and Opp. The highest correlation is between BHN and Opp; the correlation coefficient value is 0.683, which is significant at the 0.01 level. The correlation between FoW and Opp is 0.485, which is significant at the 0.05 level. Lastly, the correlation between BHN and FoW is not significant, with p-value = 0.148.

Fig. 3. Factor Correlations.

Figure 4 further shows that all the three factors correlate with both the radial and the non-radial DEA-based SPIs. An interesting observation to make, however, is that the FoW factor contributes to a greater extent to the construction of the radial DEA-based SPI (correlation coefficient = 0.872, significant at the 0.01 level), whereas in the case of the non-radial DEA-based SPI, the BHN and Opp factors are the ones contributing more towards its construction (correlation coefficients are 0.814 and 0.947, respectively, at the 0.01 level).

Figure 5 represents the correlations between the radial DEA-based SPI and the non-radial DEA-based SPI. It is to be noted that the legend of the region classifications in the graph is the same as before: 1 is the coast (represented with a circle symbol), 2 is the highlands (represented with a square symbol), and 3 is the jungle (represented with a star symbol). This

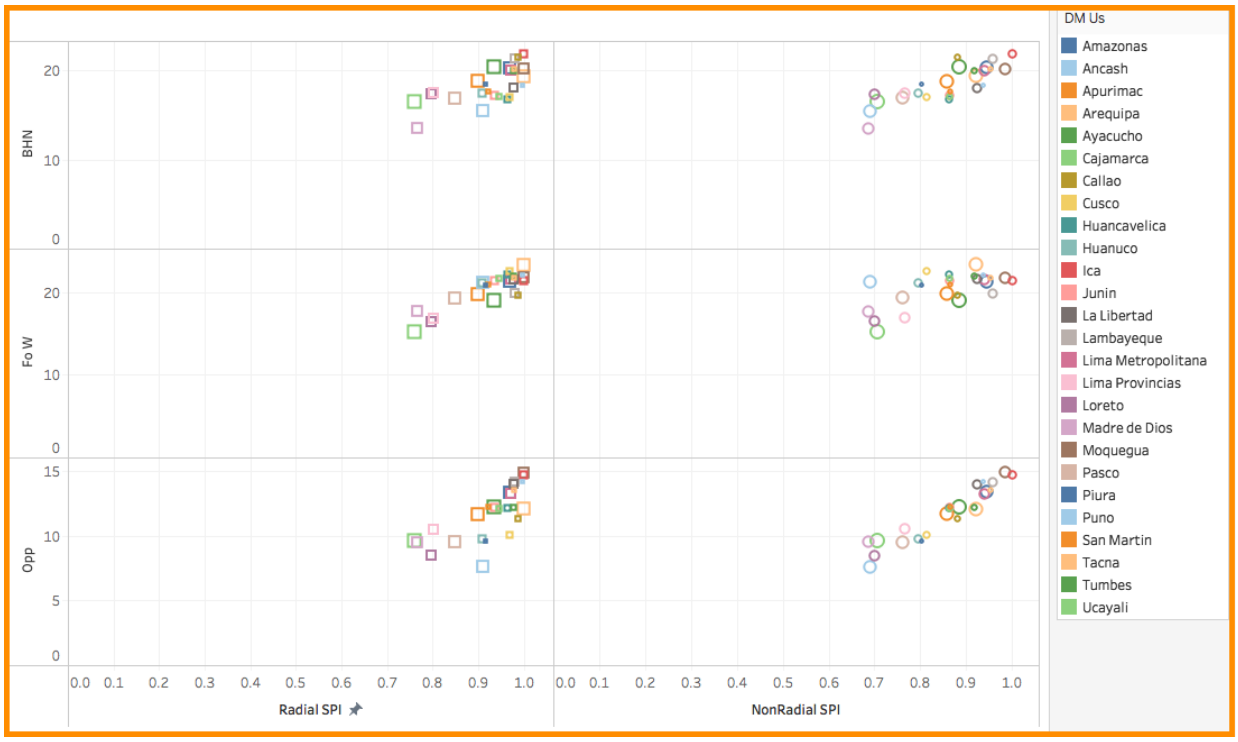

Fig. 4. Correlations between factors and radial vs. non-radial DEA-based SPI.

graph shows a positive correlation between the radial DEA-based SPI and the non-radial DEA-based SPI, with a correlation coefficient of 0.867, which is significant at the 0.01 level. The horizontal and vertical lines represent the averages of the non-radial DEA-based SPI and the radial DEA-based SPI, respectively; with the following confidence intervals: 0.8189 – 0.8948 for the non-radial DEA-based SPI and 0.8970 – 0.9568 for the radial DEA-based SPI. We also provide the trend line, represented by the dotted line, along with its 95% confidence band. We can observe that the average of the radial DEA-based SPI is higher than the average of the non-radial DEA-based SPI (0.9269 versus 0.8569). Also, the radial DEA-based SPI counts with a higher variation, which can be appreciated from the width of the confidence interval.

Based on Figure 5, we can draw some noteworthy observations. First, in the case of the non-radial DEA-based SPI, we can notice that except for one region (Lima Provincias), all the coastal regions are above the average. Similarly, in the case of the radial DEA-based SPI, we can notice that except for two regions (Lima Provincias and Tumbes), all the coastal regions are above the average yet again. This indicates that overall, coastal regions tend to be better performers in terms of social progress than the highlands and jungle regions. Further, we can notice that while in the case of the radial DEA-based SPI, there is only one highlands region (Ayacucho) situated above the average, in the case of the radial DEA-based SPI, we have four highlands regions that are above the average; this is because these four regions are performing well in a particular dimension, which "pushes" them into the high performers category. This happens because, as Charles & Diaz (2016) indicated, the non-radial DEA-based SPI values the performance in every factor, whereas the radial DEA-based index tends to focus almost exclusively on the factors in which the DMU performs better.

Fig. 5. Correlation between the radial and non-radial DEA-based SPIs.

Another observation to make is that the reqions of Ica, Moquegua, Lambayeque, Ancash, Arequipa, La Libertad, Ayacucho, and Tacna are above the averages of both the radial and non-radial DEA-based SPIs; in other words, independent of the type of DEA model used, these regions are the ones that can be said to be high performers in terms of social progress. This observation is noteworthy considering that according to Table 1, some of these regions are ranked in the middle, which would make them to be perceived as average performers instead. For example, see Ayacucho, a region that ranks 7th according to the radial DEA and 10th according to the non-radial DEA; or see La Libertad, which ranks 6th according to the radial DEA and 8th according to the non-radial DEA. Despite this, a visual representation clearly indicates that both Ayacucho and La Libertad are high performers, which points towards the cautiousness that should be attached to the interpretation of ranks, in general. Furthermore, Ayacucho is an interesting and isolated case; this is the only highlands region that is situated above the averages of both the radial and non-radial DEA-based SPIs, joining the high performers group of coastal regions.

By contrast, the worst performers are Pasco, Lima Provincias, Loreto, Ucayali, and Madre de Dios. These regions are well below the averages of both the radial and non-radial DEA-based SPIs; the reason behind their low performance lies in the fact that they each obtain rather poor scores in at least one factor of social progress. As such, Lima Provincias performs poorly in the FoW factor, Pasco in the Opp factor, Loreto and Ucayali in both FoW and Opp, and Madre de Dios in BHN and Opp. In light of this observation, concerned policymakers and regional authorities should give particular attention to these five cases, recommending local strategies,

so as to improve the social investment in the mentioned factors and thus increase the regions' social progress level (Charles & D'Alessio, 2019). San Martin, Huanuco, Puno, Amazonas, Apurimac, and Junin are also located below the mentioned averages. Nevertheless, they are within the confidence interval of the average for either the radial or the non-radial DEA-based SPI; therefore, it can be said that they are average performers in terms of social progress.

Finally, Figure 6 represents the rank correlations between the radial DEA-based SPI and the non-radial DEA-based SPI. The legend of the region classifications in the graph is the same as in Figure 5. This graph shows a positive correlation between the SPI radial rank and the SPI non-radial rank, with a correlation coefficient of 0.885, which is significant at the 0.01 level. The horizontal and vertical lines represent the medians of the SPI radial rank (median = 11.5) and the SPI non-radial rank (median = 13.5), respectively, with their median confidence intervals.

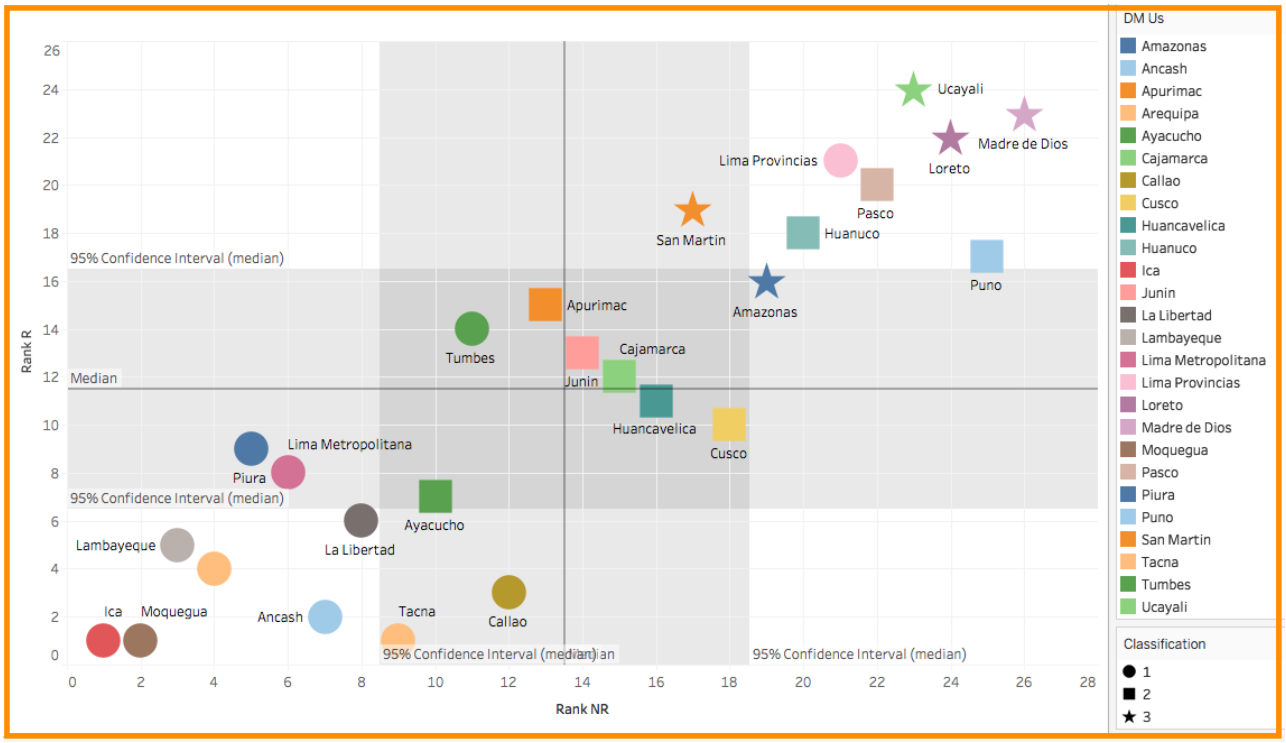

Fig. 6. Rank correlation between the radial and non-radial DEA-based SPIs.

At a closer inspection, we can observe that, similar to Figure 5, the same group of coastal regions dominate the rank (Ica, Moquegua, Lambayeque, Ancash, La Libertad, and Arequipa), with the exception of two regions, Ayacucho and Tacna, who find themselves positioned rather as average performers in view of the ranks occupied; also, we can notice that the worst perform-ers are the jungle regions. Overall, Figures 5 and 6 yield consistent insights. Perhaps the most interesting observations to make here are in relation to Lima Metropolitana and Huancavel-ica. On the one hand, Lima Metropolitana is generally ranked 1st in terms of competitiveness (CENTRUM Cat ólica Graduate Business School, 2012; Charles & Zegarra, 2014), counting with the highest GDP share (53,6%, INEI, 2009) and the second highest GDP per capita in the country (S/.12,860, according to INEI, 2009), and one of the lowest poverty rates in the coun-

try (15.4%, according to INEI, 2014). Despite this, however, Lima Metropolitana does not top the rank, being more of an average performer in terms of social progress. On the other hand, Huancavelica is generally considered to be the least competitive region in the country (Charles, 2015a; Charles, 2015b), counting with the highest poverty rate in the country (77.2%, according to INEI, 2009) and one of the lowest GDP per capita (S/.3,453, according to INEI, 2014); but then again, despite this, Huancavelica ranks as an average performer in social progress.

Another interesting case is posed by the region of Madre de Dios. This region has a relatively high GDP per capita (S/.7,555, according to INEI, 2014) and the lowest poverty rate in the country (12.7%, according to INEI, 2009); in terms of social progress, however, it is ranked 23rd in the radial DEA and 26th in the non-radial DEA, making it the worst performer among all the regions. These are fascinating cases, whose results are counter-intuitive, drawing attention to the fact that ranks and scores should always be taken with a pinch of salt. Furthermore, this further supports the necessity to develop better and more refined methodologies for the creation of regional indices for social progress, the kind of indices that can reflect to the reality with higher accuracy.

5. Conclusions

It has been the endeavour of the present manuscript to introduce an improved methodology to measure social progress at the regional level, with an application to the Peruvian regions. Our approach joins together the methodology of the Objective General Index in the first phase (Sei, 2016) with Data Envelopment Analysis in the second phase. This approach to measuring social progress is novel and represents the main contribution of the manuscript. The benefits are twofold: on the one hand, we account for the variation in the two stages of the first phase and on the other hand, we build an index based on relative measures in the second phase. The approach, however, is not without limitations. From a methodological point of view, while we do account for variation at the sub-factor level, we do not account for the same at the factor level; this is an avenue for future research. Furthermore, from a conceptual stand, the framework and variables adopted here are by no means perfect, our analysis being confined to the framework developed by Social Progress Imperative (2018). Despite this, the proposed approach is a step forward towards an improved measurement of wellbeing and social progress.

The data generated using this approach allowed us both to rank the Peruvian regions more accurately and to determine the sources of competitive strength or weakness of each region. As previously mentioned, some of the results obtained are according to expectations, especially when seen at a more 'macro' level. As such, findings suggest that the most efficient regions in terms of social progress are located on the coast, with the regions of Ancash, Arequipa, Ica, La Libertad, Lambayeque, and Moquegua forming the group of high performers in view of both radial and non-radial DEA. In a similar fashion, the worst performers are located in the jungle regions and the group is composed of Loreto, Madre de Dios, and Ucayali. The high-lands regions are generally average performers. But some other results are counter-intuitive,

see the cases of Lima Metropolitana and Huancavelica, for example. Despite being perceived as generally the best and worst regions, respectively, in terms of a variety of aspects such as competitiveness and poverty rate, these regions are both classified as rather average performers in terms of social progress.

Today, there is a plethora of metrics that have been developed to measure social progress. Seen in isolation, each indicator tells a different, yet incomplete story. Overall, there is no single methodology and no general agreement on the existence of a set of standardised or holistic indicators to measure social progress. This highlights that in order to get a more realistic picture of the wellbeing of a region, it is important to develop more accurate measures of social progress, which do not only consider the expansion of the set of variables considered in the assessment of social progress, but also integrate various methods and approaches together, refining thus, the methodology used to measure social progress. As mentioned, the latter has been the endeavour of the present manuscript.

Economic growth is, for obvious reasons, important. But if economic growth does little to improve social wellbeing, should it be a primary goal of government policy, as it keeps on being today? This continues to be a fascinating question, given its far-reaching policy implications. The only sure conclusion, as Easterlin (1974) also acknowledged, is that we need much more research on the nature and causes of human welfare. Bradburn (1969, p. 233) made a similar point when he stated that: "Insofar as we have greater understanding of how people arrive at their judgments of their own happiness and how social forces are related to those judgments, we shall be in a better position to formulate and execute effective social policies." Without much doubt, designing measurements that combine objective wellbeing data with subjective wellbeing data (Diener, 2002; Diener & Oishi, 2000) can lead to a much richer notion of a nation's wellbeing status. We position this as an avenue for future research on the topic. It is the belief of the authors that the proposed social progress index, together with the GDP and other measures of societal progress, can account for the virtuous dynamics of inclusive growth, which is essential to strengthen not only in Peru, but worldwide.

Overall, the present study has important implications for practice. Social progress and wellbeing throughout the world has arrived at a critical turning point (Estes, 2019), so providing a better snapshot of the ranking of the regions in a country in terms of social progress may help policymakers concerned with creating the conditions for nations to satisfy at least the basic social and material needs of their increasing populations (Estes & Morgan, 1976) to identify the weaknesses and strengths of each region, the main gaps, and the potential for improvement; this, in turn, could further assist them in guiding policies of social investment. Moreover, meaningful comparisons can be made among regions.

Acknowledgments

The authors would like to thank the anonymous reviewers for their valuable feedback on the previous version of this manuscript. The first author would further like to acknowledge and thank for the financial support received from the Santander Chair of Efficiency & Productiv-

ity in the Center of Operations Research (CIO), at the University Miguel Hernandez of Elche (Spain), to participate in and present the current research at the 2018 International Workshop on Efficiency and Productivity.

References

Bradburn, N. M. (1969). The structure of psychological well-being. Chicago, IL: Aldine.

CENTRUM Católica Graduate Business School. (2012). Índice de competitividad regional del Perú 2011. Lima, Peru: CENTRUM Católica Graduate Business School, PUCP.

CENTRUM Católica Graduate Business School. (2016). Índice de Progreso Social Regional Perú 2016. Lima, Peru: CENTRUM Católica Graduate Business School, PUCP & Social Progress Imperative.

Charles, V. (2015a). Mining cluster development in Peru: Learning from the International Best Practice. Journal of Applied Environmental and Biological Sciences, 5(1), 1-13.

Charles, V. (2015b). Mining cluster development in Peru: From triple helix to the four clover. Strategia, 38, 38–46.

Charles, V., & D'Alessio, F. A. (2019). An envelopment-based approach to measuring regional social progress. Socio-Economic Planning Sciences. In press.

Charles, V., & Diaz, G. (2016). A non-radial DEA index for Peruvian regional competitiveness. Social Indicators Research, 134(2), 747-770.

Charles, V., & Sei, T. (2019). A Two-Stage OGI Approach to Compute the Regional Competitiveness Index. Competitiveness Review: An International Business Journal, 29(2), 78-95.

Charles, V., & Zegarra, L. F. (2014). Measuring regional competitiveness through Data Envelopment Analysis: A Peruvian case. Expert Systems with Applications, 41, 5371-5381.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. European Journal of Operational Research, 2(6), 429-444.

Cobb, C. W., Halstead, T., & Rowe, J. (1995). If GDP is up why is America down?. Atlantic Monthly, 276(4), 59-78.

Davies, W. (2015). Spirits of neoliberalism: "Competitiveness" and "Wellbeing" indicators as rivals of worth. In R. Rottenburg, S. E. Merry, S.-J. Park., & J. Mugler (Eds.), The World of Indicators: The Making of Governmental Knowledge through Quantification (pp. 283-306). Cambridge, UK: Cambridge University Press.

Diener, E. (2002). "Will money increase subjective well-being?". Social Indicators Research, 57, 119-169.

Diener, E., & Oishi, S. (2000). Money and happiness: Income and subjective well-being across nations. In E. Diener & E. Suh (Eds.), Subjective well-being across cultures (pp. 185-218). Cambridge: MIT Press.

Easterlin, R. A. (1974). Does Economic Growth Improve the Human Lot? Some Empirical Evidence. In P. A. David & M. W. Reder (Eds.), Nations and Households in Economic Growth. Essays in Honor of Moses Abramovitz (pp. 89-125). New York, NY: Academic Press, Inc.

Estes, R. J. (2019). The Social Progress of Nations Revisited. Social Indicators Research, 144, 539-574.

Estes, R. J., & Morgan, J. S. (1976). World social welfare analysis: A theoretical model. International Social Work, 19(2), 29–41.

European Commission (2007). Beyond GDP. Measuring progress, true wealth, and the well-being of nations. Retrieved from http://www.beyond-gdp.eu/index.html

INEI. (2009). Censos Nacionales de Población y Vivienda. Lima, Peru: National Institute of Statistics and Information of Peru.

INEI. (2014). Censos Nacionales de Población y Vivienda. Lima, Peru: National Institute of Statistics and Information of Peru.

Kubiszewski et al. (2013). Beyond GDP: Measuring and achieving global genuine progress. Ecological Economics, 93, 57-68.

Kuznets, S. (1934). National Income, 1929–1932. Cambridge, MA: National Bureau of Economic Research.

Lovell, C. A. K., & Pastor, J. T. (1999). Radial DEA models without inputs or without outputs. European Journal of Operational Research, 118(1), 46-51.

Marshall, & A. W., Olkin, I. (1968). Scaling of matrices to achieve specified row and column sums. Numerische Mathematik, 12, 83-90.

Porter, M. E., Stern, S., & Green, M. (2016). Social Progress Index 2016. Washington, DC: Social Progress Imperative.

Sei, T. (2016). An objective general index for multivariate ordered data. Journal of Multivariate Analysis, 147, 247–264.

Social Progress Imperative. (2018). 2018 Social Progress Index. Retrieved from http://www.socialprogressimperative.org/

Stiglitz, J. E., Sen, A., & Fitoussi, J. P. (2009). Report by the commission on the measurement of economic performance and social progress. Retrieved from: https://ec.europa.eu/eurostat/documents/118025/118123/Fitoussi+Commission+report

Stiglitz, J. E., Sen, A., & Fitoussi, J. P. (2010). Mismeasuring Our Lives: Why GDP Doesn't Add Up. New York, NY: The New Press.