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Active vibration control using mechanical and electrical analogies

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Abstract. Mechanical-electrical analogous circuit models are widely used in electromechanical system design as they represent the function of a coupled electrical and mechanical system using an equivalent electrical system. This research uses electrical circuits to establish a discussion of simple active vibration control principles using two scenarios: an active vibration isolation system and an active dynamic vibration absorber (DVA) using a voice coil motor (VCM) actuator. Active control laws such as gain scheduling are intuitively explained using circuit analysis techniques. Active vibration control approaches are typically constraint by electrical power requirements. The electrical analogous is a fast approach for specifying power requirements on the experimental test platform which is based on a vibration shaker that provides the based excitation required for the single Degree-of-Freedom (1DoF) vibration model under study.

1   Introduction

Dynamic Vibration Absorbers (DVAs) were patented by Herman Frahzm in 1909 [1] and the principles of passive DVAs design were fully described by Ormondroyd and Den Hartog [2, 3].

DVAs has been extensively studied in building structures, as defence mechanism against earthquakes, to counter seismic movements and wind forces [4]. DVAs are widely used to control vibration in structures. Passive control and active control are the typical methods for mitigating undesired vibrations. Active control requires sensors and actuators and electronic control systems to reduce vibration levels [5].

Coupled electrical and mechanical systems are typically analysed using equivalent circuit models. Equivalent circuit representations are widely used in electroacoustic [6] and electromechanical transducer analysis [7] due to their powerful visualization of the underlying physical phenomena. This study analyses an active DVA and an active vibration isolation system using electrical analogous circuit models.

2   Modelling the experimental active vibration control rig.

The experimental rig shown in Figure 1a. was built to test the concept of active control using an adjustable mass connected to a planar spring and one VCM actuator. Vibration measurements are performed using two accelerometers, one connected to the absorber mass and the other one to the shaker table. Two electrical signals are recorded for VCM characterization that correspond to the VCM current and voltage which are measured using one isolated differential voltage probe and one current probe respectively. The physical model which is a pictorial representation of the system under study is shown in Figure 1b.

The vibration model corresponds to a 1DoF with base excitation as shown in Figure 2.

The differential equation that describes the motion of the mass 'm' using 2nd Newton's law is shown in Equation 1.

$$m\ddot{x}_1 = U - k(x_1 - x_2) - c(\dot{x}_1 - \dot{x}_2) \tag{1}$$

where x_2 is the base excitation which is provided by the shaker table , x_1 describes the spring suspended mass motion, k is the helical spring stiffness, c is the mechanical damping and U is a generic actuator force.

Rig specifications are shown in Table 1. The VCM is a GVCM-051-051-01 manufactured by Moticont. VCM impedance measurements were taken using the frequency response analyser PSM1735 with an IAI LCR interface manufactured by Newtons4th.
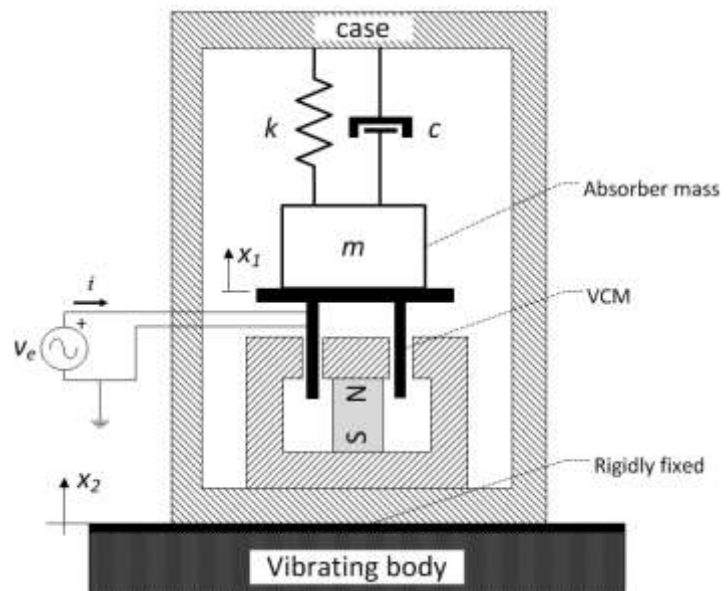
a) **Experimental rig**   b) **Physical model**

Figure 1. Active vibration control rig

Table 1 Test rig parameters

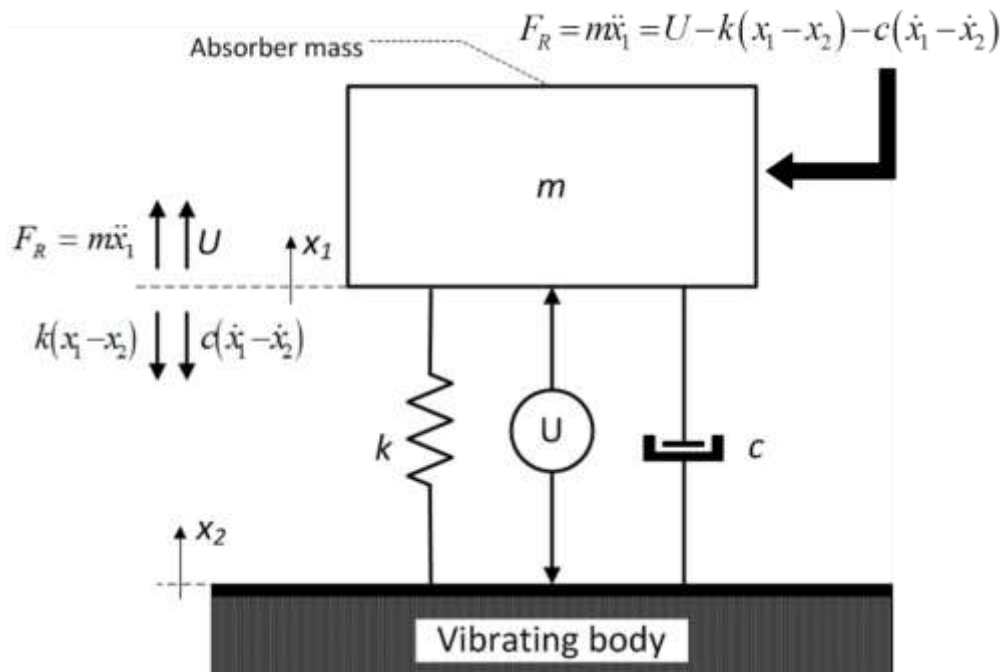| Parameter	Value	Unit	Parameter	Value	Unit
m	432.4	[g]	$L_E\big|_{f=50Hz}^{\text{no magnet air coil}}$	2.1	[mH]
c	0.5 Negligible	[Ns/m]	$R_E\big|_{f=50Hz}^{\text{no magnet air coil}}$	3.08	[Ω]
k	42.167	[kN/m]	$L_E\big|_{f=50Hz}^{\text{with magnet}}$	4.1	[mH]
$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	49.7	[Hz]	$R_E\big|_{f=50Hz}^{\text{with magnet}}$	3.3	[Ω]
			$\alpha_{VCM}\big|_{datasheet}$	8	[N/A]

Figure 2. Active control vibration model with a generic actuator U (1DoF)

2.1   Equivalent circuit representation of the 1DoF active control rig with a generic actuator

An analogous electrical and mechanical system will have differential equations of the same form. There are two analogues for the system shown in Figure 2. Type I analogues use current as mechanical force representation and Type II analogues use voltage as force representation. Table 2 shows the relationship between electrical and mechanical quantities.

Table 2 Interpretation of electrical and mechanical quantities

Electrical Quantity	Mechanical Analog I (Force-Current)	Mechanical Analog II (Force Voltage)
Voltage, e	Velocity, v	Force, f
Current, i	Force, f	Velocity, v
Resistance, R	Lubricity, 1/B (Inverse friction)	Friction, B
Capacitance, C	Mass, M	Compliance, 1/K (Inverse spring constant)
Inductance, L	Compliance, 1/K (Inverse spring constant)	Mass, M

Type I is represented in Figure 3 whilst Type II in Figure 4. A circuit analysis using Kirchoff's Laws yields the same differential equation as shown in Equation 1. For Type I, the differential equations are obtained using KCL in node A as shown in Equation 2 and for Type II, they are obtained using KVL in loop I as shown in Equation 3.

$$\left(\text{KCL N}_\text{A} \right) \; C\frac{d\dot{x}_1}{dt} + \frac{\left(\dot{x}_1 - \dot{x}_2 \right)}{R} + \frac{1}{L}\int \left(\dot{x}_1 - \dot{x}_2 \right)dt = u$$

$$\left(\text{KCL N}_\text{A} \right) \; m\ddot{x}_1 + c\left(\dot{x}_1 - \dot{x}_2 \right) + k\left(x_1 - x_2 \right) = u \tag{2}$$

initial conditions

$$U\left(0s \right) = 0 \; ; \; x_1\left(0s \right) = x_1\left(0s \right) = 0$$

$$\left(\text{KVL Loop 1} \right) \; u + \frac{1}{C}\int \left(\dot{x}_2 - \dot{x}_1 \right)dt + R\left(\dot{x}_2 - \dot{x}_1 \right) - L\frac{d\dot{x}_1}{dt} = 0$$

$$\left(\text{KVL Loop 1} \right) \; u + k\left(x_2 - x_1 \right) + c\left(\dot{x}_2 - \dot{x}_1 \right) - m\ddot{x}_1 = 0 \tag{3}$$

initial conditions

$$U\left(0s \right) = 0 \; ; \; x_1\left(0s \right) = x_1\left(0s \right) = 0$$
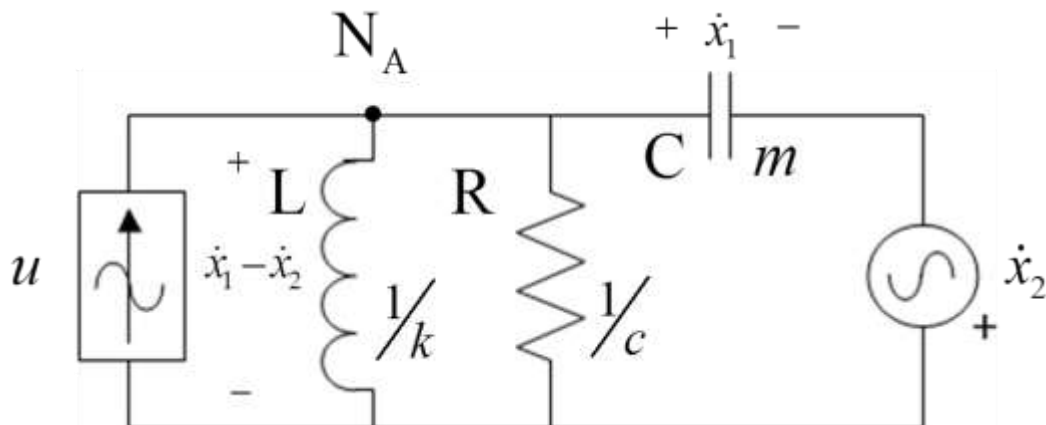

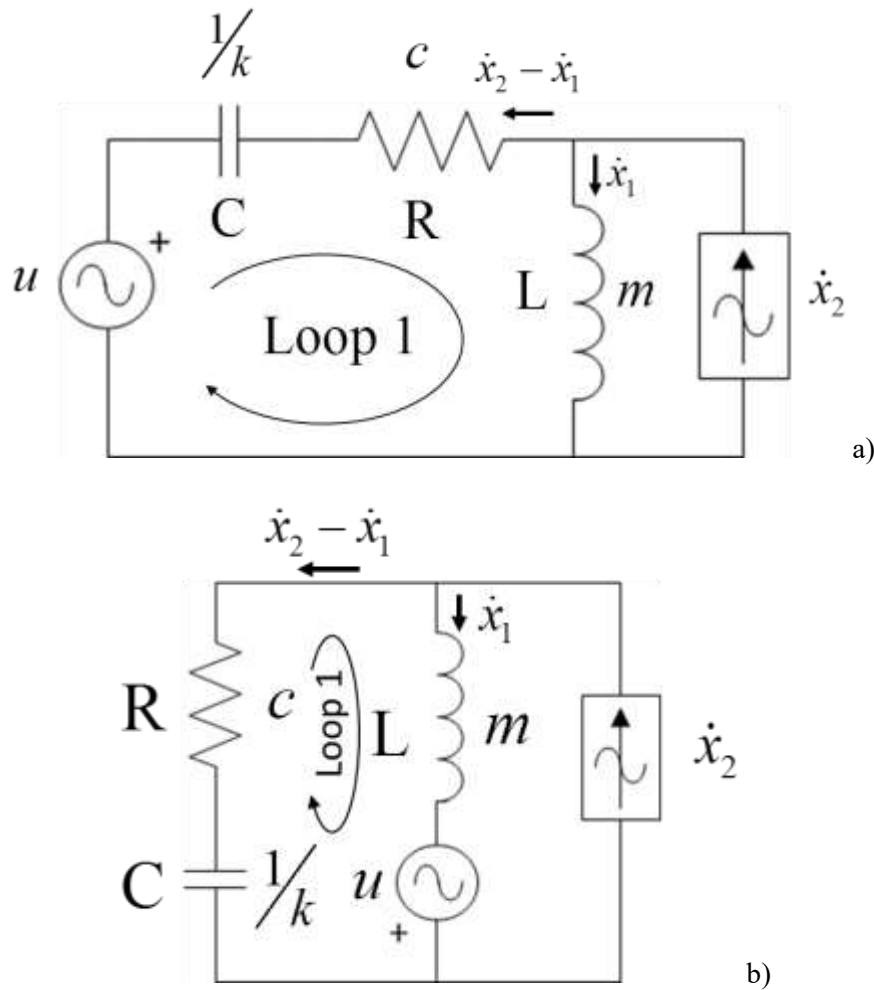**Figure 3. 1DoF active control rig with generic actuator (Type I)**

Figure 4. 1DoF active control rig with generic actuator (Type II)

2.2   Active vibration control strategies for two real scenarios: vibration isolation and vibration absorbers.

The concept of vibration isolation is that the mass 'm' is held motionless. That implies that the speed $\dot{x}_1$ must be 0. Using the circuit shown in Figure 4, vibration isolation implies that the current $\dot{x}_1$ must be 0. Using the simplified circuit in Figure 5 the control law could be derived by using superposition theorem and the current divider rule as shown in Equation 4. Electrical variables such as current and voltages are expressed as forces and velocities respectively.

$$(sX_1) = (sX_2)\frac{Z_T}{Z_{MP}} + \frac{U}{Z_{MS}+Z_{MP}} = (sX_2)\frac{Z_{MS}}{Z_{MS}+Z_{MP}} + \frac{U}{Z_{MS}+Z_{MP}}$$

(4)

If we want to minimize x1 motion → $\dot{x}_1 = 0$ then $U = -(sX_2)\cdot Z_{MS}$

$$Z_{MS}(s) = c + \frac{k}{s}$$

(5)

$$U(s) = -(sX_2) \cdot Z_{MS} \rightarrow U(s) = -(cs+k)X_2(s)$$

The amplitude of an undamped DVA tends to be infinite at its resonant frequency. The optimum active control laws could be derived from the circuit in Figure 5. The actuator force u must be chosen for an infinite actuator stroke at the excitation frequency of $\dot{x}_2$. An active DVA that mimics a passive DVA without damping at any frequency implies an actuator speed or current $\dot{x}_1 - \dot{x}_2$ of infinite value. This condition is easily obtained using Kirchoff's Voltage Laws after applying the source transformation theorem as shown in Figure 6a. and Equation 6.

$$\dot{x}_1 - \dot{x}_2 = \infty \text{ only if } U + s(X_2 - X_1)Z_{MS} + s(X_2 - X_1)Z_{MP} = 0$$

$$U(s) = -(X_2(s) - X_1(s))(ms^2 + cs + k), \text{ if } s = j\omega$$

(6)

$$U(j\omega) = -(X_2(j\omega) - X_1(j\omega))(c + j\omega(m-k)) \cdot (j\omega) = -(X_2(j\omega) - X_1(j\omega))\left(c + j\left(\omega m - \frac{k}{\omega}\right)\right) \cdot (j\omega)$$

If we use the circuit in Figure 6b. the condition is achieved in Equation 7.

$$\dot{x}_1 = \infty \text{ only if } -U + (sX_1) \cdot (Z_{MS} + Z_{MP}) = 0$$

$$U(s) = X_1(s)(ms^2 + cs + k)$$

(7)

$$U(j\omega) = X_1(j\omega) \cdot (c + j\omega(m-k)) \cdot (j\omega) = X_1(j\omega)\left(c + j\left(\omega m - \frac{k}{\omega}\right)\right) \cdot (j\omega)$$

According to Equation.6 and Equation 7, the actuator force must remove the damping of the system and it should provide a force proportional to actuator relative position $(x_1 - x_2)$ of magnitude α or acceleration $(\ddot{x}_1 - \ddot{x}_2)$ of magnitude γ. Gain scheduling control consists on two possible gains, one proportional to acceleration (α) and the other proportional to position (γ) are defined in Equation 7 using the natural resonant frequency of the absorber mass [8].

$$U(j\omega) = -(X_2(j\omega) - X_1(j\omega))\left(c + j\omega\underbrace{\left(m - \frac{k}{\omega^2}\right)}_{\alpha}\right) \cdot (j\omega) = -(X_2(j\omega) - X_1(j\omega))\left(c + \frac{j}{\omega}\underbrace{(\omega^2 m - k)}_{\gamma}\right) \cdot (j\omega)$$

Using the natural resonant frequency $\omega_0 = \sqrt{\frac{k}{m}}$

(9)

(Acceleration gain) $\alpha = \left(m - \frac{k}{\omega^2}\right) \xrightarrow{\omega_0 = \sqrt{\frac{k}{m}}} \alpha = m\left(1 - \left(\frac{\omega_0}{\omega}\right)^2\right)$

(Position gain) $\gamma = (\omega^2 m - k) \xrightarrow{\omega_0 = \sqrt{\frac{k}{m}}} \gamma = m(\omega^2 - \omega_0^2)$
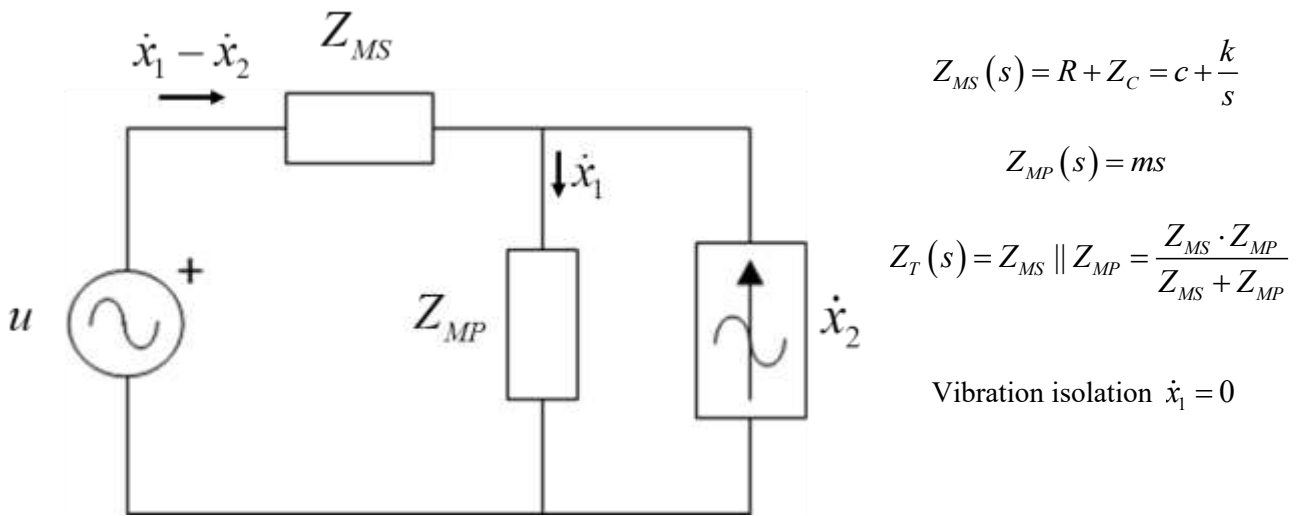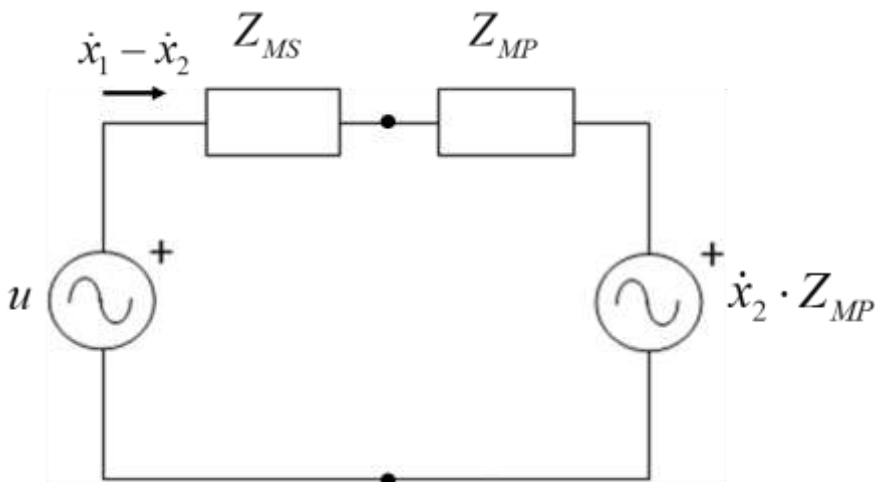
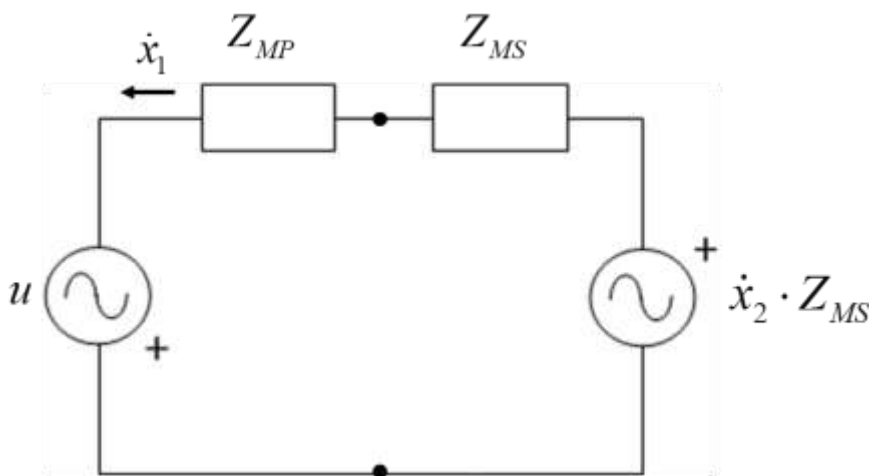$$Z_{MS}(s) = R + Z_C = c + \frac{k}{s}$$

$$Z_{MP}(s) = ms$$

$$Z_T(s) = Z_{MS} \| Z_{MP} = \frac{Z_{MS} \cdot Z_{MP}}{Z_{MS} + Z_{MP}}$$

Vibration isolation $\dot{x}_1 = 0$

Figure 5. Actuator force for active vibration isolation

Active DVA.

Using circuit in Fig 4a

$\dot{x}_1 - \dot{x}_2 = \infty$ at resonance when
$\dot{x}_2 \neq 0$

a)

Active DVA

Using circuit in Fig 4b

$\dot{x}_1 = \infty$ at resonance when
$\dot{x}_2 \neq 0$

b)

Figure 6. Actuator force for an Active DVA

2.3   Equivalent circuit representation of the 1DoF active control rig with a VCM

If the chosen actuator is a Voice Coil Motor, the coupling equation between the electrical and mechanical system is due to Lorentz Force and Faradays's Law of Induction. The actuator mechanical force is proportional to actuator's current if there is a constant magnetic flux density in the VCM gap for all possible actuator stroke positions $N\dfrac{d\Phi}{dx} = cte$. The Back Electromotive Force is proportional to the total number of turns and actuator speed as shown in Equation 9.

α_{VCM} is the motor constant

$$\alpha_{VCM} = N\frac{d\Phi}{dx} = Bl \quad \text{where:}$$

      N is the number of turns,

      B the magnetic flux density in the gap   (9)

      l, the length of the wounded wire

(Actuator mechanical force) $F = Bl \cdot I = \alpha_{VCM} \cdot l$

(Induced voltage BEMF) $E = -N\dfrac{d\Phi}{dt} = -N\dfrac{d\Phi}{dx}\dfrac{dx}{dt} = -\alpha_{VCM}\dot{x}$

A possible circuit representation that couples the mechanical system and electrical side uses a gyrator and it is shown in Figure 7. The governing differential equations for this circuit are shown in Equation 10.
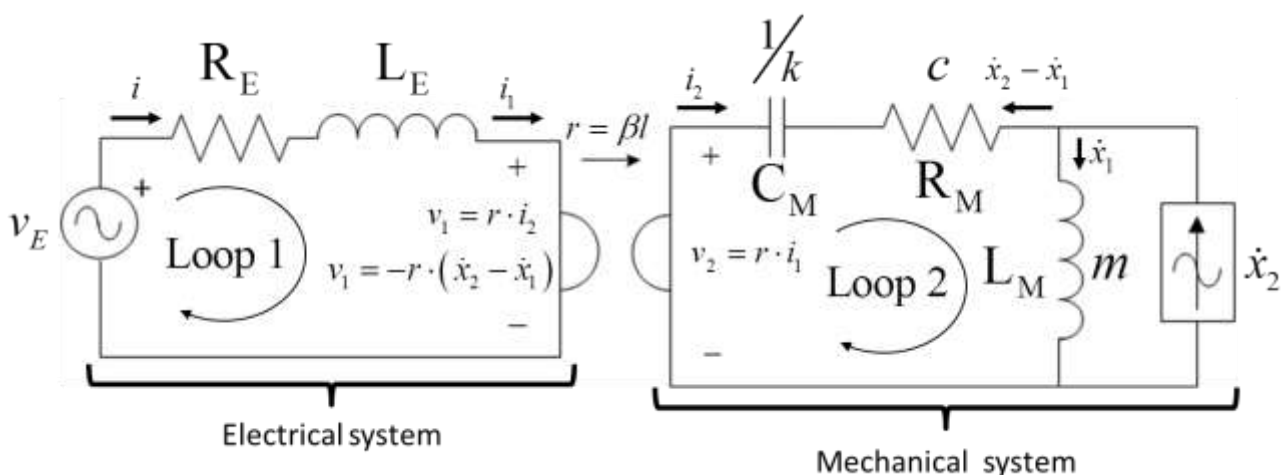

Figure 7. 1DoF active control rig with VCM (Type II)

$(\text{KVL Loop 1}) \ V_E - R_E \cdot i - L_E \cdot \dfrac{di}{dt} - v_1 = 0$ (10)

$$\left(\text{KVL Loop 2}\right)\ v_2 + k\left(x_2 - x_1\right) + c\left(\dot{x}_2 - \dot{x}_1\right) - m\ddot{x}_1 = 0$$

$$r = \beta l = \alpha_{VCM} \quad \begin{cases} V_E - R_E \cdot i - L_E \cdot \dfrac{di}{dt} + r \cdot \left(\dot{x}_2 - \dot{x}_1\right) = 0 \\[2mm] r \cdot i_1 + k\left(x_2 - x_1\right) + c\left(\dot{x}_2 - \dot{x}_1\right) - m\ddot{x}_1 = 0 \end{cases}$$

For active vibration isolation or active control of a DVA, the VCM current should be same as the control laws shown in Equations 5-7 respectively. For example, ideal vibration isolation requires a VCM current according to Equation 11. As it can be seen, this control strategy only requires reading the shaker table position, speed or acceleration ($x_2, \dot{x}_2, \ddot{x}_2$). External electronics such as linear servo motor drives could adjust the VCM voltage to ensure the VCM current follows the desired control law.

$$u = \alpha_{VCM} \cdot i = \dot{x}_2 \cdot Z_{MS} \rightarrow I\left(s\right) = \left(sX_2\right) \cdot \frac{Z_{MS}}{\alpha_{VCM}} \rightarrow I\left(s\right) = \left(\frac{c}{\alpha_{VCM}} s + \frac{k}{\alpha_{VCM}}\right) X_2\left(s\right) \tag{11}$$

2.4 Electrical impedance and power requirements

Electrical power requirements for active vibration control are one of the most important factors when choosing between Passive and Active solutions for mitigating vibrations in an industrial application. The circuit shown in Figure 8 corresponds to the equivalent impedance seen by the electrical system due to the mechanical one after impedance transformations.
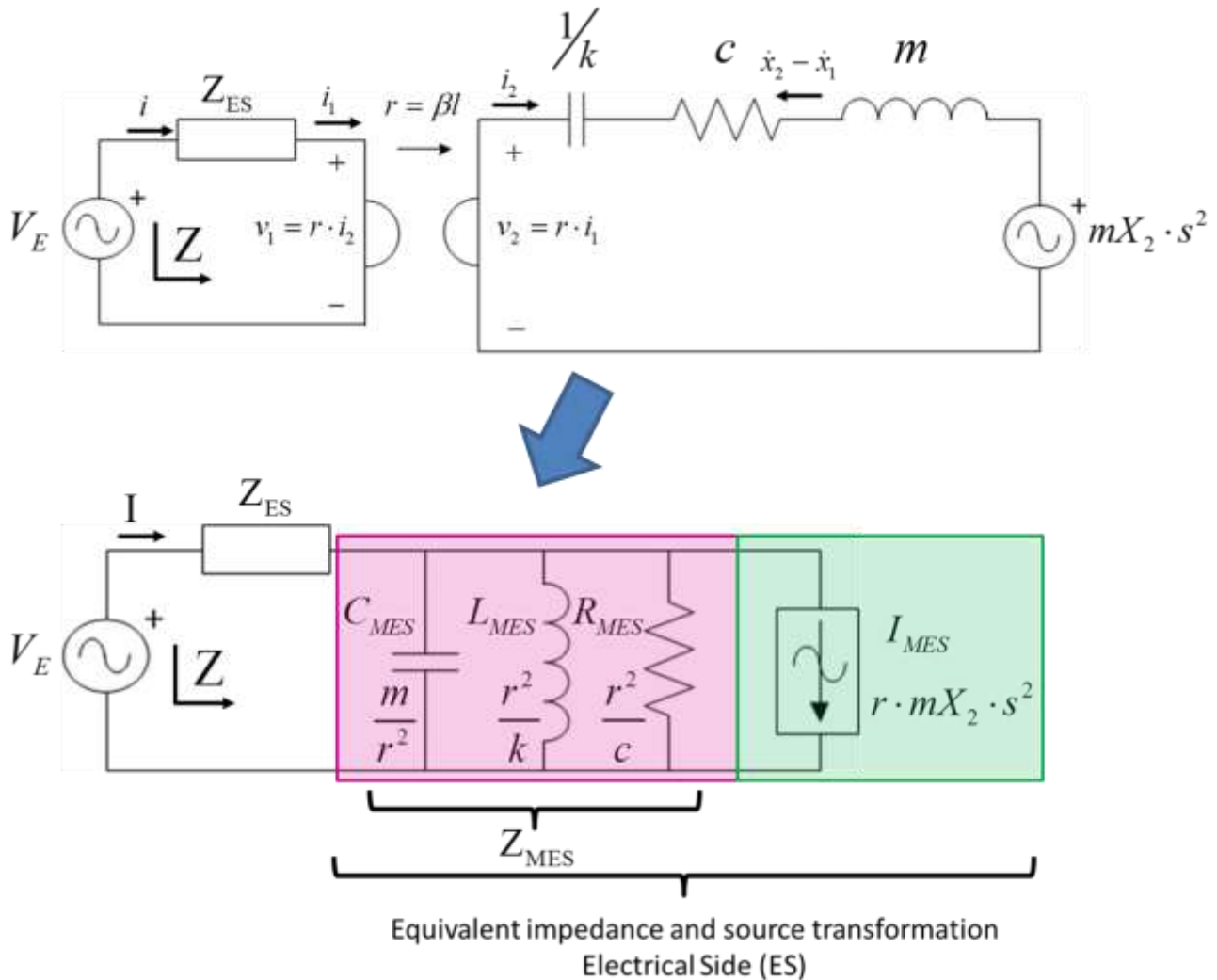
Figure 8. VCM Electrical impedance (Z)

An analysis of Figure 8 using source transformation theorem yields to the following electrical impedance expression as shown in Equation 12.

$$Z(s) = \frac{V_E(s) + I_{MES}(s)\ Z_{MES}(s)}{Z_{ES}(s) + Z_{MES}(s)}$$

where

(12)

$$Z_{MES}(s) = \frac{1}{\dfrac{1}{R_{MES}} + \dfrac{1}{s \cdot L_{MES}} + \dfrac{1}{1/(s \cdot C_{MES})}} \quad \text{and} \quad Z_{ES}(s) = R_E + s \cdot L_E$$

If the shaker table does not move, the impedance equation could be further simplified as shown in Equation 13.

$$Z(s) = \frac{V_E(s)}{Z_{ES}(s) + Z_{MES}(s)} \quad \text{when} \quad X_2(s) = 0 \tag{13}$$

Electrical power requirements could be calculated using the impedance expressions as shown in Equation 14.

$$P_{ES}(s) = Z(s) \cdot I^2(s) = \frac{V_E^2(s)}{Z(s)} \begin{cases} \mathrm{Re}\{P_{ES}(s)\} \to \text{Active power} \\ \mathrm{Im}\{P_{ES}(s)\} \to \text{Reactive power} \end{cases} \tag{14}$$

2.5 Mass on the spring motion using the equivalent circuit

Using the circuit transformation shown in Figure 9 an equivalent circuit as shown in Figure 10 could be analysed for obtaining the mass motion $x_1, \dot{x}_1, \ddot{x}_1$
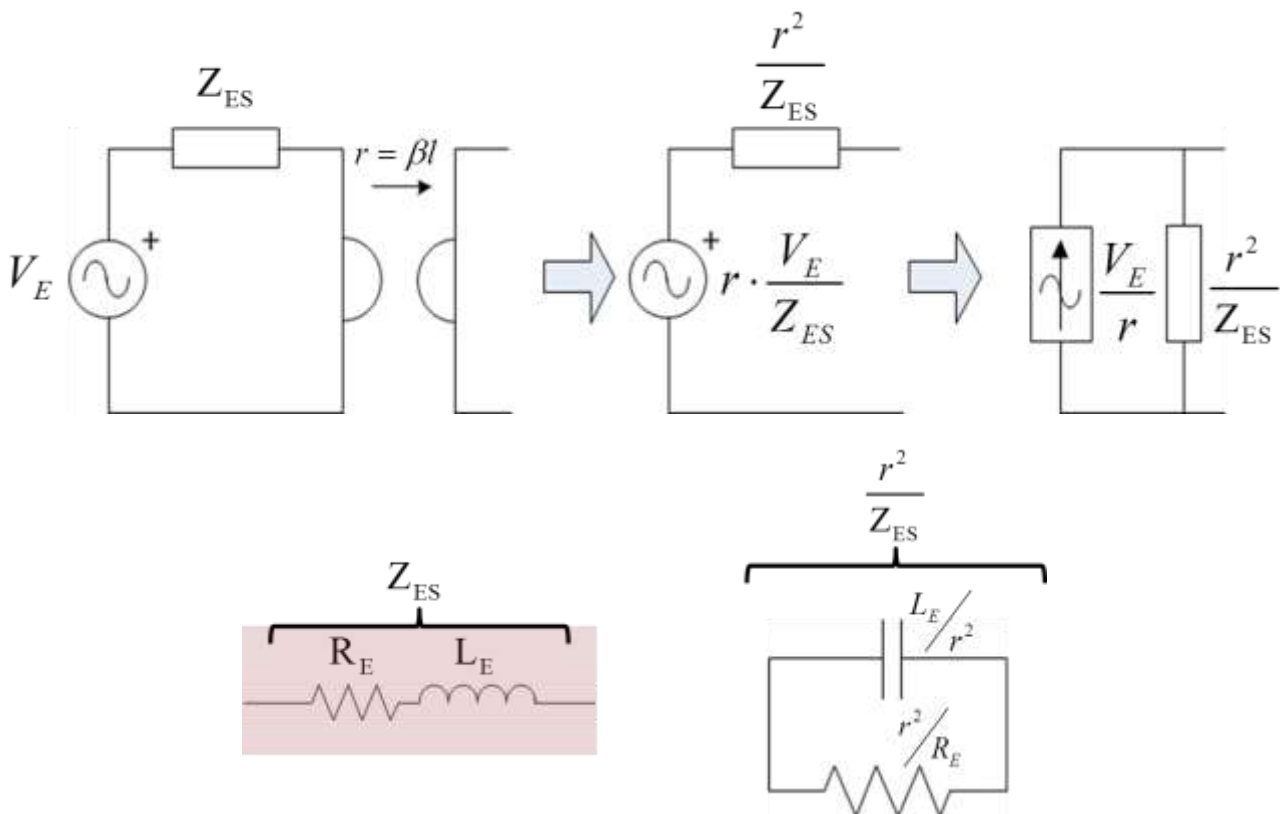
Figure 9. Equivalent circuit transformation to the Mechanical System
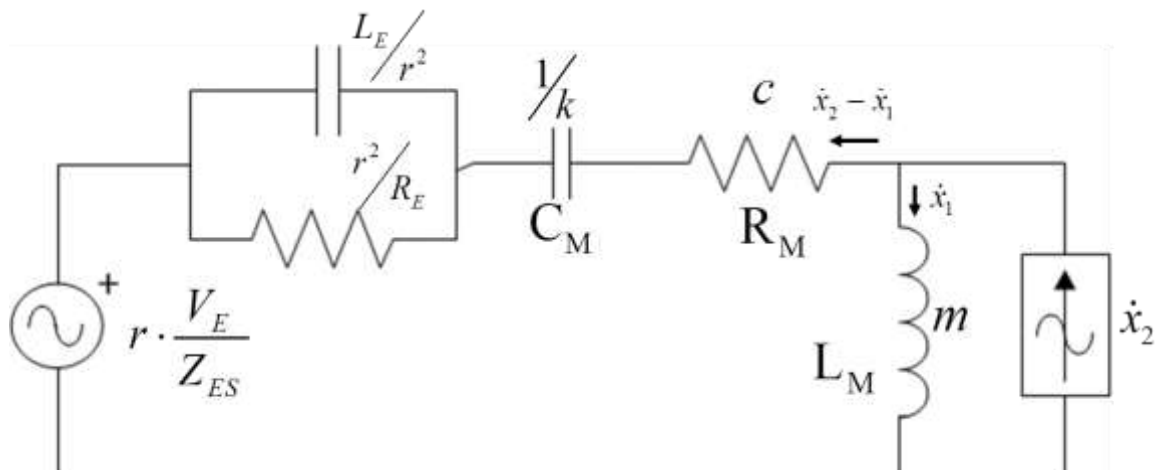
Figure 10. Equivalent circuit for absorber motion determination

The absorber mass motion could be analysed using Superposition Theorem and the current divider rule according to Equation 15.

$$\dot{x}_1 = \dot{x}_2 \frac{Z_T}{Z_{L_M}} + \frac{r \cdot \dfrac{V_E}{Z_{ES}}}{\dfrac{r^2}{Z_{ES}} + R_M + Z_{C_M} + Z_{L_M}} \quad \text{where}$$

$$Z_T = \frac{1}{\dfrac{r^2}{Z_{L_M}} + \dfrac{1}{\dfrac{1}{Z_{ES}} + R_M + Z_{C_M} + Z_{L_M}}} \tag{15}$$

$$Z_{ES}(s) = R_E + s \cdot L_E \; ; \; R_M = c \; ; \; Z_{LM} = ms \; ; \; Z_{CM} = \frac{k}{s}$$

The circuit shown in Figure 10 describes the shift in resonant frequency when a short circuit is placed across the VCM terminals.

3   Model validation

Transmissibility experiments were carried out with the VCM terminals in open and closed circuit respectively. The model in Figure 7 could consider the added damping due to eddy current losses in the VCM actuator when there is an open circuit across the VCM terminals as show in Figure 11.

Fitting transmissibility plots to experimental results using the theoretical model in Equation 15 shows that the best damping factor is 6.1 N/(m/s) when the test was performed with an open circuit across VCM terminals and the damping factor is 20.2 N/(m/s) when a short circuit is placed across VCM terminals, Figures 12-13. Under short circuit test the resonant frequency shifted from 49.7Hz to 51Hz. This shift in frequency is explained in Figure10 as additional stiffness added by the VCM inductance (L_E).

$$\frac{X_2}{X_1} = \frac{\left(ms^2 + cs + k\right)}{\left(cs + k\right)} \tag{16}$$

The open circuit resistor value in this case is calculated according to Equation 17 if r is 8 N/A and $c\big|_{oc} = 6.1$ Ns/m and c=0.5Ns/m, therefore R_{OC} is 11.4Ω.

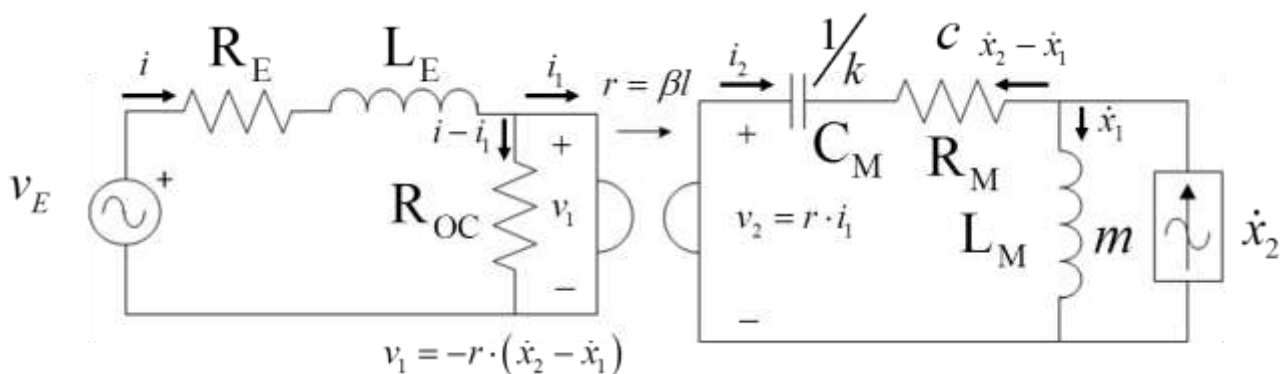$$R_{OC} = \frac{r^2}{c\big|_{oc} - c} \tag{17}$$

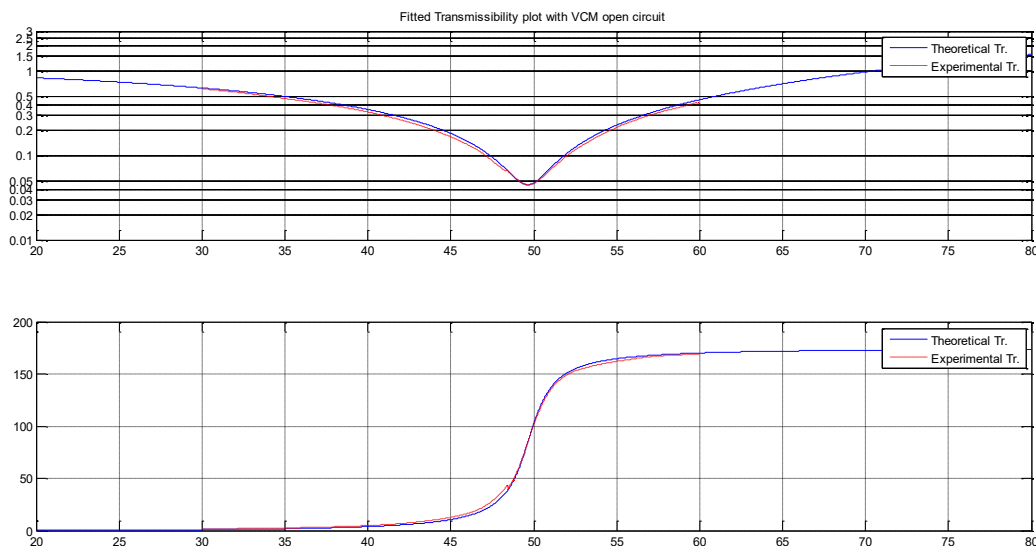**Figure 11. Model improvements to include VCM Open Circuit damping**

Figure 12. Transmissibility plots with an open circuit across VCM terminals

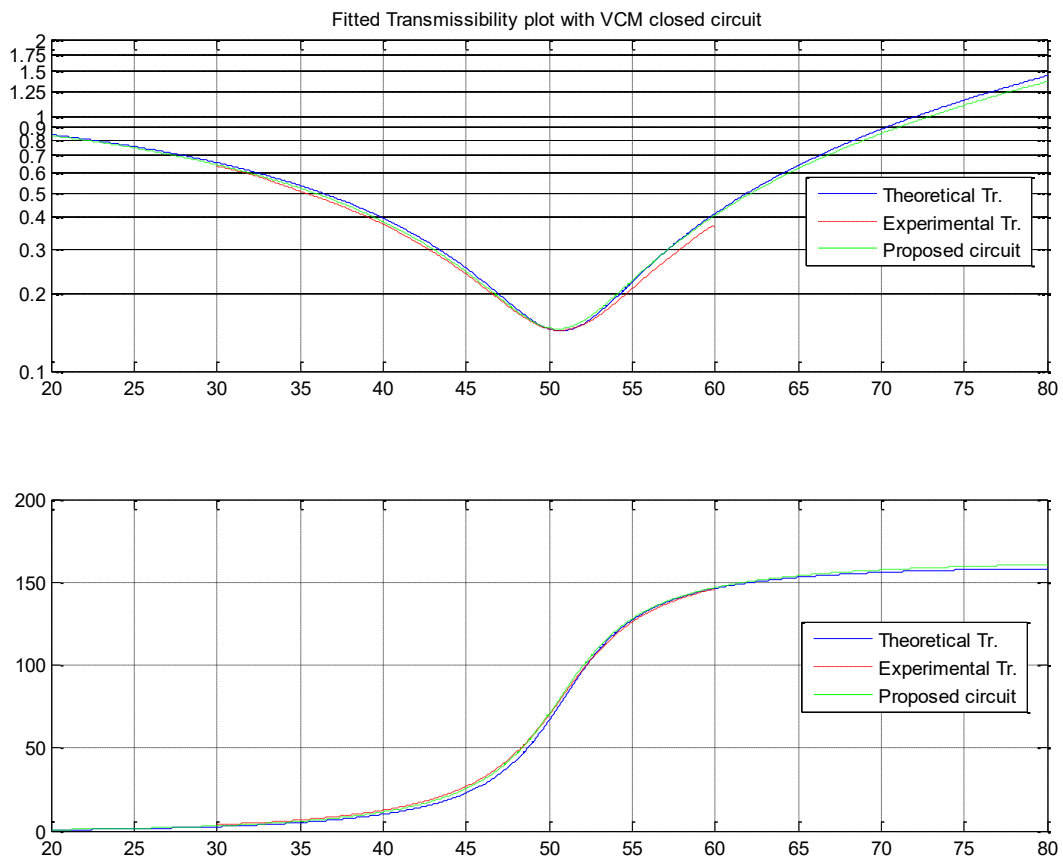Fitted Transmissibility plot with VCM closed circuit

Figure 13. Transmissibility plots with a closed circuit across VCM terminals

The circuit shown in Figure 11 provides a trend that is in good agreement with experimental results when the VCM terminals are under open and closed circuit conditions respectively. The green trace in Figure 13 corresponds to the circuit values shown in Table 3.

Table 3 Circuit values

Parameter	Value	Unit	Parameter	Value	Unit
	432.4	[g]	L_E	4.1	[mH]
	0.5 Negligible	[Ns/m]	R_E	3.3	[Ω]
m	42.167	[kN/m]	$\alpha_{VCM} = r$	7.4	[N/A]
c	49.7	[Hz]	R_{OC}	9.78	[Ω]
k			C_M	23.71	[μF]
$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$			L_M	0.4324	[H]
			R_M	0.5	[Ω]

4   Conclusion

Coupled mechanical-electrical systems could be analysed with electromechanical analogous circuit models. The practicality of this method is that it provides a better visualization and interpretation of the system. A complete derivation of active control laws for dynamic vibration absorbers and vibration isolation is straightforward by analysing these circuits. Electrical power requirements could be defined in the frequency domain by calculating the electrical impedance seen by the input voltage source connected to the VCM. The proposed equivalent circuits show a good agreement with experimental results.

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