Nonlinear Vibrations of a Cable System with a Tuned Mass Damper under Deterministic and Stochastic Base Excitation

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Abstract

This paper investigates a dynamic model of a cable – mass system equipped with an auxiliary mass element to act as a transverse tuned mass damper (TMD). The cable length varies slowly while the system is mounted in a vertical host structure swaying at low frequencies. This results in base excitation acting upon the cable - mass system. The model takes into account the fact that the longitudinal elastic stretching of the cable is coupled with their transverse motions. The TMD is applied to reduce the dynamic response of the system. The parameters of TMD are selected by the application of a linearized model and a single-mode approximation. In this approach the excitation is represented as a narrow-band Gaussian process mean-square equivalent to a harmonic process. The deterministic model and stochastic model are used to predict and control the resonance response of the system.

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Keywords: Cable System; Slowly Varying; Vibrations; Tuned Mass Damper

1. Introduction

Moving cable systems are deployed in many engineering systems. In some applications the length of cables vary during operation rendering the system non-stationary. For example, in hoist, elevator and mine lifting installations the payload- carrying cables moving at speed within a host structure have time-variant length and the natural frequencies vary with their length [1]. The modular cable - mass installations are mounted within host structures that

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are often subject to environmental phenomena such as wind and seismic excitations [2]. The corresponding response and excitation mechanisms can be represented by deterministic functions or treated as stochastic processes [3,4]. In this paper a deterministic model and the corresponding stochastic model of a mass –cable system constrained to move vertically in a host structure are considered. The system is equipped with an auxiliary spring – damper - mass combination attached to the main (primary) mass to act as a *tuned mass damper* (TMD). In this arrangement the TMD can be applied to mitigate the effects of resonance when the frequency of the base motion becomes near the natural frequency corresponding to the primary mass – cable mode.

2. Mathematical model

2.1. System Configuration

Fig. 1 shows a mass – vertical cable system mounted within a host structure with a primary mass M attached to the lower end of the cable of time-varying length $L = L(t)$ moving axially at transport speed V. The cable is mounted within a host structure of height $AB = Z_0$ with its upper end passing through O at the top of the structure. The mean quasi-static tension, mass per unit length, modulus of elasticity and cross-sectional metallic area of the cable are denoted as $T^i = \left[M + m_d + m(L-x)\right](g-a)$, m, E and A, respectively. The Eulerian spatial coordinate x is measured from the upper end downwards as shown. The lateral dynamic displacements of the cable are denoted as $v(x,t)$. They are coupled with the longitudinal vibrations denoted as $u(x,t)$. The mass M is constrained in the lateral direction by a linear spring of coefficient of stiffness k and can move in the vertical direction. Its lateral and longitudinal vibrations are denoted as $v_M(t)$ and $u_M(t)$, respectively. An auxilliary small mass m_d is attached to the main mass via a spring – dashpot system of coefficient of stiffness k_d and coefficient of viscous damping c_d, respectively. The auxilliary mass is constrained to move horizontally with its motion denoted as z_d. The equations of motion Eq. (1) are developed by applying the extended Hamilton's principle.

$$
\begin{aligned}
& m\frac{D^2u}{Dt^2} - EA\varepsilon_x = 0, \quad m\frac{D^2v}{Dt^2} - Tv_{xx} + m(g-a)(xv_{xx}+v_x) - EA(\varepsilon v_x)_x = 0 \\
& M\ddot{v}_M + T^i(L)v_x\big|_{x=L} + k\Delta - k_d(z_d - v_M) - c_d(\dot{z}_d - \dot{v}_M) + EA\varepsilon\big|_{x=L} v_x\big|_{x=L} = 0 \\
& m_d\ddot{z}_d + k_d(z_d - v_M) + c_d(\dot{z}_d - \dot{v}_M) = 0, \quad (M+m_d)\ddot{u}_M + EA\varepsilon\big|_{x=L} = 0
\end{aligned} \tag{1}
$$

where $\varepsilon = u_x + v_x^2/2$ represents the axial strain, $D(\,)/Dt = (\,)_t + V(\,)_x$, $(\,)_t$ and $(\,)_x$ represent partial derivatives with respect to time t and x, respectively, and $T = (M + m_d + mL)(g-a)$, where a represents the acceleration of the transport motion. For tensioned members such as metallic cables the lateral frequencies are much lower than the longitudinal frequencies. Thus, considering that the excitations frequencies are much lower than the fundamental longitudinal frequencies the longitudinal inertia of the cable can be neglected in the first equation in (1). Thus, this equation can be integrated to give $u_x = e(t) - v_x^2/2$ where $e(t)$ represents the quasi-static axial strain in the cable.

2.2. Base Excitation

The host structure is subjected to bending deformations acting as base excitation and described by the polynomial shape function $\Psi(\eta) = 3\eta^2 - 2\eta^3$ (see Fig. 1), where $\eta = z/Z_0$ with z denoting a coordinate measured from ground level and Z_0 representing the height at the top end of the cable. In this scenario the structure undergoes harmonic motions $v_0(t)$ of frequency Ω_0 and amplitude A_0, measured at the level Z_0. Thus, at the upper end the displacements of the cable are $v(0,t) = v_0(t)$. In order to accommodate the base excitation in the equations of motion (1) the overall lateral displacements of the cable – mass system are expressed by Eq. (2).

$$
v(x,t) = \underline{v}(x,t) + \left(1 + \frac{\Psi_L - 1}{L}x\right)v_0(t), \quad \Psi_L = \Psi\left(\frac{Z_0 - L}{Z_0}\right) \tag{2}
$$

It is assumed that the variation of length L with time is small. Thus, L is a slowly varying function in time meaning that the change of $L(t)$ over a period corresponding to the fundamental frequency of the system is small compared to L [3]. In order to represent this fact a slow time scale defined as $\tau = \epsilon t$, where where $\epsilon \ll 1$ is a small parameter, is introduced. This parameter is quantified as $\epsilon = V/(\omega_0 L_0)$ where ω_0 denotes the lowest natural frequency and L_0 is the corresponding length of the cable [5]. Considering that $L = L(\tau)$ the relative lateral displacements are then expressed using the finite series given by Eq. (3).

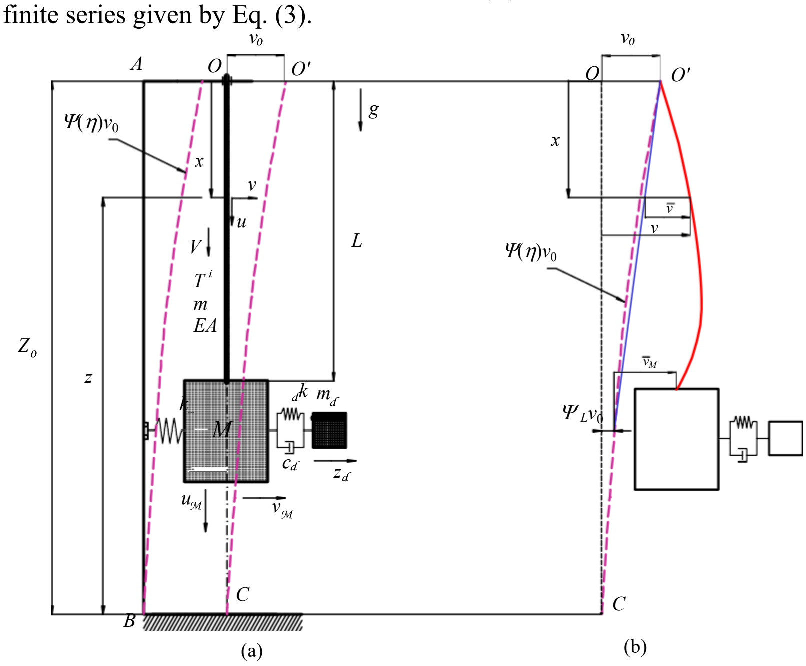

Fig. 1. Model of the mass – cable system under consideration: (a) undeformed configuration, (b) deformed configuration.

$$v(x,t;\tau) = \sum_{n=1}^{N} \Phi_n\left[x; L(\tau)\right] q_n(t) \tag{3}$$

where $\Phi_n\left[x; L(\tau)\right]$ are orthogonal trial functions depending on the spatial coordinate and are varying slowly with the length of the cable. The trial functions satisfy the homogenous boundary conditions and are defined as $\Phi_n\left[x; L(\tau)\right] = \sin\left[\lambda_n(L(\tau))x\right]$, $n = 1, 2, \ldots, N$, with N denoting the number of terms/ modes taken in (3). The slowly varying eigenvalues $\lambda_n(\tau)$ are defined by the frequency equation given as

$$\left(k - \frac{M}{m} T_{Md} \lambda_n^2\right) \sin(\lambda_n L) + T_{Md} \lambda_n \cos(\lambda_n L) = 0, \quad T_{Md} \equiv T^i(L) = (M + m_d)(g - a) \tag{4}$$

The generalised coordinates $q_n(t)$ are time-dependent and fast varying. Using (2), (3) together with (4) in the equations of motion, orthogonalising with respect to the trial functions, when terms $O(\epsilon)$ and $O(\epsilon^2)$ are neglected, equations Eq. (5) and Eq. (6) result.

$$m_r \ddot{q}_r + k_r q_r + \sum_{n=1}^{N} K_{rn} q_n - \left[k_d\left(z_d - \bar{v}_M\right) c_d\left(\dot{z}_d - \dot{\bar{v}}_M\right)\right] \Phi_r(L) - EA\bar{e}\left[\sum_{n=1}^{N} \Gamma_{rn} q_n - \frac{\Psi_L - 1}{L} \Phi_r(L) v_0\right] = Q_r$$
$$m_d \ddot{z}_d + k_d\left(z_d - \bar{v}_M\right) + c_d\left(\dot{z}_d - \dot{\bar{v}}_M\right) = Z_d, \quad (M + m_d)\ddot{u}_M + EA\bar{e}(t;\tau) = 0, \tag{5}$$

$$\bar{v}_M = \sum_{n=1}^{N} \Phi_n\left[L(\tau)\right] q_n(t),\; Q_r = \beta_r^{(0)}(\tau) v_0(t) + \beta_r^{(1)}(\tau)\dot{v}_0(t) + \beta_r^{(2)}(\tau)\ddot{v}_0(t),\; Z_d = k_d\Psi_L^0 v_0 + c_d\Psi_L^0 \dot{v}_0,$$
$$K_{rn} = m\left[g\Psi_{rn} + (g-a)\left(\Theta_{rn} - L\Upsilon_{rn}\right)\right],\; \Psi_{rn} = \int_0^L x\Phi_n''\Phi_r\,dx, \Psi_{rn} = \int_0^L \Phi_n'\Phi_r\,dx,\; \Upsilon_{rn} = \int_0^L \Phi_n'\Phi_r\,dx,\; k_r = m_r\omega_r^2,$$
$$\omega_r = \sigma_r\sqrt{\frac{T_{Md}}{m}},\; e = \frac{u_M}{L(\tau)} + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{2L(\tau)}\kappa_{ij}(\tau)q_i q_j + v_0(t)\frac{\Psi_L - 1}{L^2(\tau)}\sum_{i=1}^{N}\alpha_i(\tau)q_i + \frac{1}{2}\left(\frac{\Psi_L - 1}{L(\tau)}\right)^2 v_0^2(t),\tag{6}$$
$$m_r = \int_0^L \Phi_r^2\,dx + M\Phi_r^2(L),\; \kappa_{ij} = \int_0^L \Phi_i'\Phi_j'\,dx,\; \alpha_i = \Phi_i(L),\; \Gamma_{rn} = \Upsilon_{rn} - \Phi_n'(L)\Phi_r(L),$$

2.3. Single-mode approximation

Consider a single-mode approximation and the rth mode with the relative displacements expressed as $v(x,t;\tau) = \Phi_r\left[x, L(\tau)\right]q_r(t)$. The equations of motion (5) are then expressed as (7), where $\tilde{k}_r = k_r + K_{rr}$, $\beta_r^{(0)}$, $\beta_r^{(1)}$, $\beta_r^{(2)}$ are known slowly varying coefficients and the modal damping is introduced through the coefficient $c_r = 2m_r\zeta_r\omega_r$, $\omega_r = \sqrt{k_r/m_r}$. The linearized lateral response (uncoupled from the longitudinal mode) of the main mass can then be defined by a set of two equations (8).

$$m_r\ddot{q}_r + c_r\dot{q}_r + \tilde{k}_r q_r - \left\{k_d\left[z_d - \Phi_r(L)q_r\right] + c_d\left[\dot{z}_d - \Phi_r(L)\dot{q}_r\right]\right\}\Phi_r(L) - EA\bar{e}_r\left[\Gamma_{rr}q_r - \frac{\Psi_L - 1}{L}\Phi_r(L)v_0\right] = Q_r,$$
$$m_d\ddot{z}_d + k_d\left[z_d - \Phi_r(L)q_r\right] + c_d\left[\dot{z}_d - \Phi_r(L)\dot{q}_r\right] = Z_d(t;\tau),\; (M + m_d)\ddot{u}_M + EA\bar{e}_r = 0\tag{7}$$

$$\begin{aligned} &m_r\ddot{q}_r + c_r\dot{q}_r + \tilde{k}_r q_r - \left\{k_d\left[z_d - \Phi_r(L)q_r\right] + c_d\left[\dot{z}_d - \Phi_r(L)\dot{q}_r\right]\right\}\Phi_r(L) = Q_r,\\ &m_d\ddot{z}_d + k_d\left[z_d - \Phi_r(L)q_r\right] + c_d\left[\dot{z}_d - \Phi_r(L)\dot{q}_r\right] = Z_d \end{aligned}\tag{8}$$

3. Stochastic excitation model

The motion $v_0(t)$ of the host structure is seldom exactly harmonic. For example, the excitation due to the action of wind is usually a wide-band stochastic process. Then the response in the fundamental mode is a narrow-band process with a centre frequency equal to the fundamental natural frequency Ω_0. The stochastic motion could be determined from the analysis of the structure response. Alternatively, it may be assumed that $v_0(t)$ is a narrow-band process mean-square equivalent to the harmonic process with the amplitude A_0 and the frequency Ω_0. The motion $v_0(t)$ must be continuous together with its first and second derivatives $\dot{v}_0(t)$ and $\ddot{v}_0(t)$. These conditions are satisfied by assuming that the motion $v_0(t)$ is the response of the second order auxiliary filter to the process $X(t)$, which is in turn the response of the first-order filter to the Gaussian white noise $\xi(t)$ excitation [6]. The governing equations are

$$\ddot{v}_0(t) + 2\zeta_f\Omega_0\dot{v}_0(t) + \Omega_0^2 v_0(t) = X(t);\; \dot{X}(t) + \alpha X(t) = \alpha\sqrt{2\pi S_0}\,\xi(t)\tag{9}$$

where damping ratio ζ_f of the filter defines its band width, α is the filter variable, S_0 is the constant level of the power spectrum of the white noise. Consider the linearized single-mode approximation (8). The augmented state vector defined as $\mathbf{Y}^T(t) = [q_r, \dot{q}_r, z_d, \dot{z}_d, v_0, \dot{v}_0, X]$ is then governed by the following set of stochastic equations

$$d\mathbf{Y}(t) = \mathbf{A}\mathbf{Y}(t)dt + \mathbf{b}dW(t)\tag{10}$$

where $W(t)$ is the standard Wiener process (corresponding to the Gaussian white noise $\xi(t)$) and $\mathbf{A}$ is the state matrix defined in terms of the system coefficients. The differential equations governing the second-order statistical moments of the state vector $\mathbf{Y}(t)$, i.e. the covariance matrix $\mathbf{R_{YY}} = E[\mathbf{YY}^T]$ are then represented by Eq. (11).

$$\frac{d}{dt}\mathbf{R}_{\mathbf{YY}} = \mathbf{A}\mathbf{R}_{\mathbf{YY}} + \mathbf{R}_{\mathbf{YY}}\mathbf{A}^T + \mathbf{b}\mathbf{b}^T \tag{11}$$

4. Parametric case study

A parametric study has been conducted which involves the primary mass M = 6768 kg constrained in the horizontal direction by a spring of constant k = 2.8 kN/m, suspended on n_r = 6 steel wire ropes. The ropes have mass per unit length m_r = 2.18 kg/m and longitudinal stiffness EA = 22.889 MN/m^2, each. In the scenario considered the system is ascending from the lower level, when the initial length of the ropes is $L(0)$ = 258.66 m, upwards at speed of 2.5 m/s. The travel height is 200 m so that the length of the ropes changes from $L(0)$ to L_{min} = 58.66 m during the travel. The height of the host structure is Z_0 = 261.86 m. The host structure is subjected to the fundamental resonance sway of frequency Ω_0 = 0.6597 rad/s (0.105 Hz) and the amplitude of the sway at the top level (corresponding to Z_0) is A_0 = 0.1 m. In this example a TMD system of mass ratio $\mu = m_d / m_{re} = 0.05$ and the optimum damping ratio determined as $\zeta_{op} = 0.13$ is considered to mitigate the effects of transition through the first (fundamental) lateral mode resonance of the mass-cable system. The frequency of base excitation becomes tuned to the fundamental mode during the travel when the length of the suspension ropes L is approximately 161 m (see Fig. 2 (a)). It should be noted that the fundamental longitudinal frequencies (determined as $\omega_M = \sqrt{k_{eq} / m_{eq}}$, where $k_{eq} = n_r EA / L$, $m_{eq} = M + n_r m_r L / 3$ and shown vs. L in Fig. 2(b)) are of over one order of magnitude higher than the fundamental lateral frequencies. Fig. 2(c) shows the lateral response $v_{\bar{M}}$ of the primary mass vs. time, determined by numerical simulation of nonlinear model Eq. (7), where r = 1 is used and the damping ratio $\zeta_1 = 0.1$ is assumed. The response plots with the TMD action (red line) and the response without TMD being applied (black line) are superimposed on each other, demonstrating that the fundamental mode resonance oscillations are becoming attenuated (the largest amplitude is reduced by about 34%). The corresponding longitudinal motions u_M, coupled with the lateral mode, are shown in Fig. 2(d). It is evident that the longitudinal response, which is three orders of magnitude smaller than the lateral response, is attenuated by the action of TMD on the lateral mode. Fig. 2(e) shows that in the resonance region the lateral response determined by the numerical simulation of linear model (8) (blue dashed line) is almost identical to the response determined from the nonlinear model (7) (red line). The linearized approximation (8) has been used to develop Eq. (11) to study the effects of stochastic excitation on the behaviour of the system. Fig. 3 shows the variance functions $\sigma^2_{\bar{v}_M}$ for speeds of 1.5, 2.5 and 3.5 m/s, respectively. It is evident that the higher the speed the lower the scatter levels of the response.

Concluding remarks

The proposed mathematical model accommodates the nonlinear effects of cable stretching and is used to determine the response of the system under the excitation caused by low frequency sway motions of the host structure. The lateral response of the system is then approximated by a single-mode formulation. In this approach the mode corresponding to the main mass motion should be chosen in order to implement the TMD action. The approximation is used to implement stochastic excitation model. In this model the excitation is represented as a narrow-band Gaussian process mean-square equivalent to a harmonic process. The proposed linear approximation then leads the determination of covariance matrix with its elements showing the statistical scatter of the response of the system. The dynamic behaviour of the system can readily be investigated by the application of numerical techniques. The case study presented in the paper demonstrates the effectivness of the proposed modelling approach.

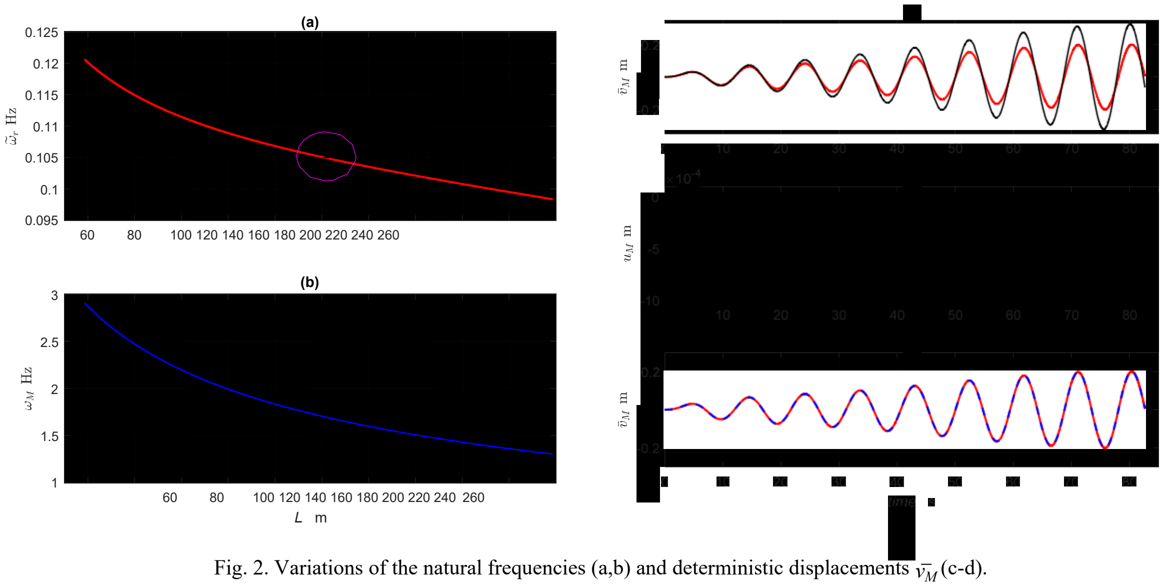

Fig. 2. Variations of the natural frequencies (a,b) and deterministic displacements $\bar{v}_M$ (c-d).

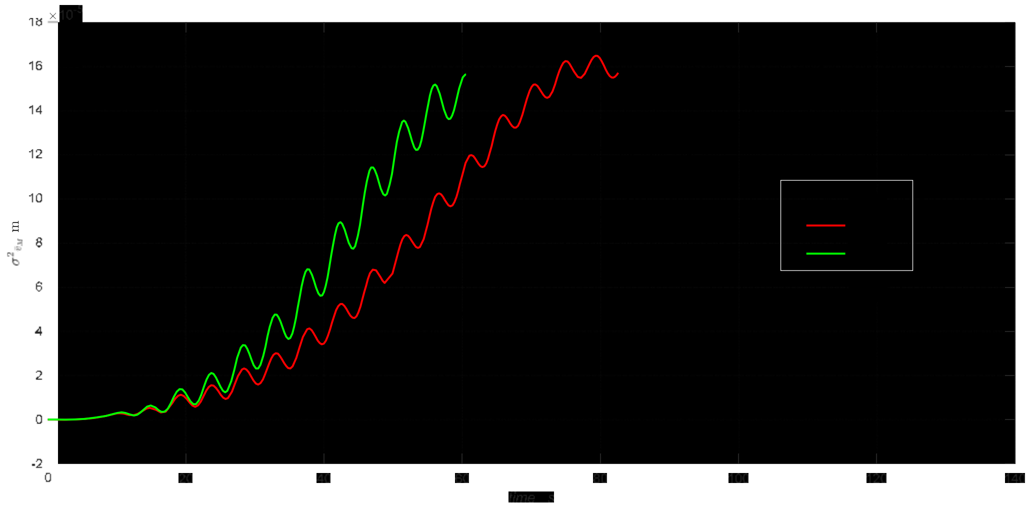

Fig. 3.Variance $\sigma^2_{\bar{v}_M}$ for speeds 1.5, 2.5 and 3.5 m/s.

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