Direct data-transformation calculation of Standardised Precipitation Indices.

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Introduction

Standardised Precipitation Indices (SPIs), a form of Drought Index, were first proposed by McKee, Doesken and Kleist in 1993. In using SPIs calculated according to their original specification, we observed that SPI-sets for UK precipitation data in general are negatively skewed and have non-zero means and non-unity standard deviations, i.e. are not standard-normally distributed. We attribute these observations to the equiprobability mapping between the cumulative Gamma distribution, used to fit the precipitation data, and the cumulative standard normal distribution, from which the SPIs are derived as abscissae.

We present a new method for calculating SPIs. The resulting sets of SPIs are significantly closer to standard-normally distributed, having (very close to) zero skewness, zero mean and unity standard deviations. The resulting root-normal distributions are, in general, also (marginally) better fits to the data than the gamma distribution used by McKee *et al.*

Hypothesis

In essence... If precipitation data are (truncated) square- or cube- root normally distributed, then SPIs can be calculated directly from the positive square- or cube- roots of the data without the need for an equiprobability mapping.

Extending this... If precipitation data are more generally (truncated) n^{th}-root normally distributed, then SPIs can be calculated directly from the positive n^{th} roots of the data without the need for an equiprobability mapping.

Square-root normally distributed data

One of the data-sets (see Method), May data for Oxford, was acceptably square-root normally distributed (actually 1.9964-root normally distributed).

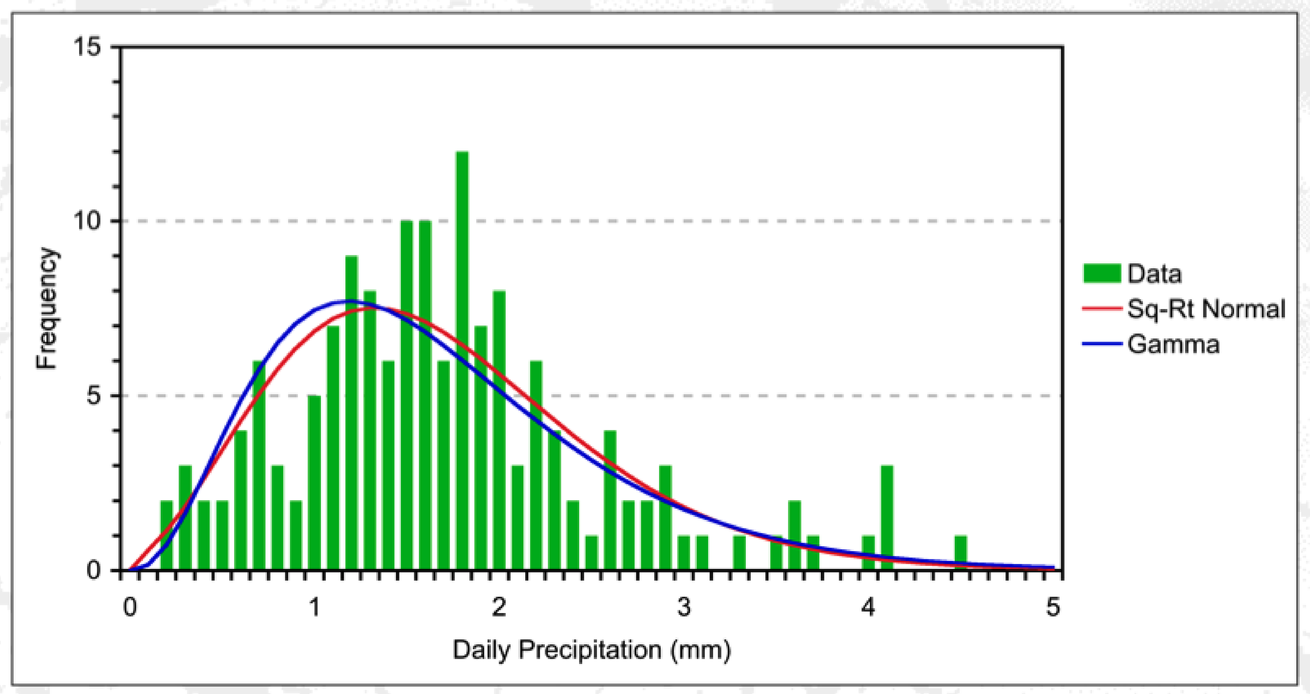

Fig 1. Square-root normally distributed data. Square-root normal and Gamma distributions also shown.

Goodness-of-fit, as measured by the correlation of the cumulative square-root normal and gamma distributions to the data, are 0.9984 and 0.9973 respectively.

SPIs were calculated according to McKee *et al.*'s specification and directly from the square-roots of the data. The distributions of these SPI-sets are compared in Table 1 and Figure 2.

Table 1. SPI distribution parameters.

	Sq-Rt	McKee *et al.*
Mean	0.0000	0.0032
St Dev	1.0000	1.0009
Skew	0.0203	-0.3594

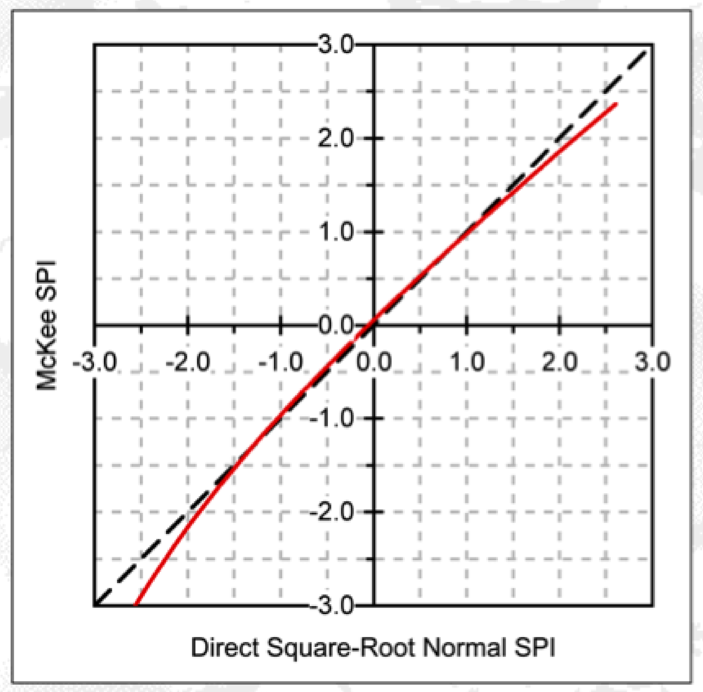

Fig 2. McKee *et al.* vs. direct square-root normal SPIs.

Method

Six UK locations having the longest precipitation records (from a larger set of locations) formed the focus for this initial investigation; Armagh (137 years), Bradford (93 years), Durham (124 years), Oxford (151 years), Sheffield (121 years) and Southampton (145 years).

Two sets of code were written in Scilab:
- to calculate SPIs according to McKee *et al.*'s specification;
- to calculate SPIs directly from n^{th}-root normally transformed data.

The algorithm for finding the optimum n^{th}-root is iterative, using the Shapiro-Francia test for normality at each iteration. It terminates with an error of n ±0.0001, i.e. at 4 decimal places, 5 significant figures, within an interval [1,10].

Results

SPIs were calculated using both methods for periods of 1, 2, 3, 4, 6, 8, 9 and 12 months for the six locations. The results are summarised in Figures 3 and 4 (for 1, 3, 6 and 12 month periods, for clarity).

These figures show the variation in mean and skewness for five intervals of widths ±1, ±2, ±3, ±4 and ±5 deciles about the median.

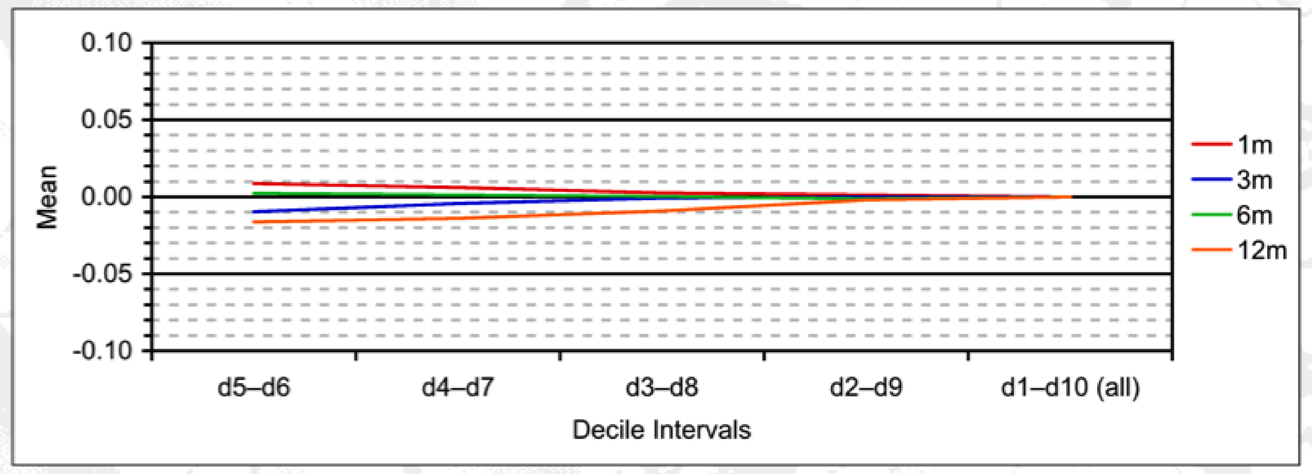

Fig 3a. Variation of SPI means, root-normal SPIs.

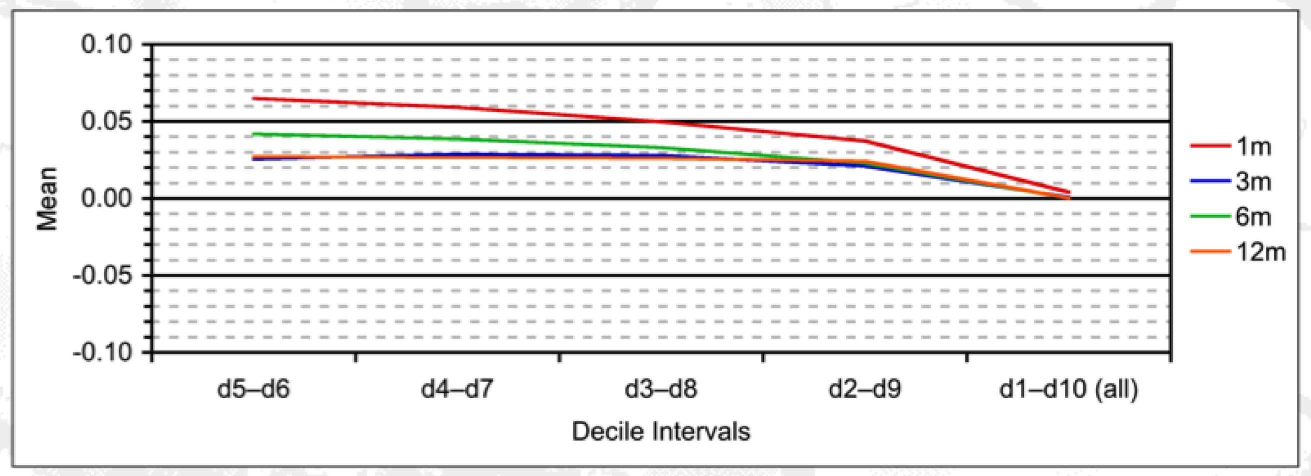

Fig 3b. Variation of SPI means, McKee *et al.* SPIs.

Discussion

The root-normal SPIs show a higher degree of consistency across the five intervals than do the McKee *et al.* SPIs. In particular, the McKee *et al.* SPIs show a distinct decrease in skewness from the central ±1 decile interval to the whole SPI set at the ±5 decile interval.

This confirms the initial observations which stimulated the investigation. For any given set of cumulative probabilities, there is a nonlinear relationship between sets of abscissae calculated according to gamma-distributions and root-normal distributions.

The extent to which the nonlinearities inherent between direct and equiprobability-mapped SPIs are significant depends on the data and the detail of the interpretation. However, as shown in Figure 2 and confirmed for the majority of other data investigated (>90%), SPIs according to McKee *et al.*'s specification tend to over-index drought (negative SPIs) and under-index anti-drought (positive SPIs).

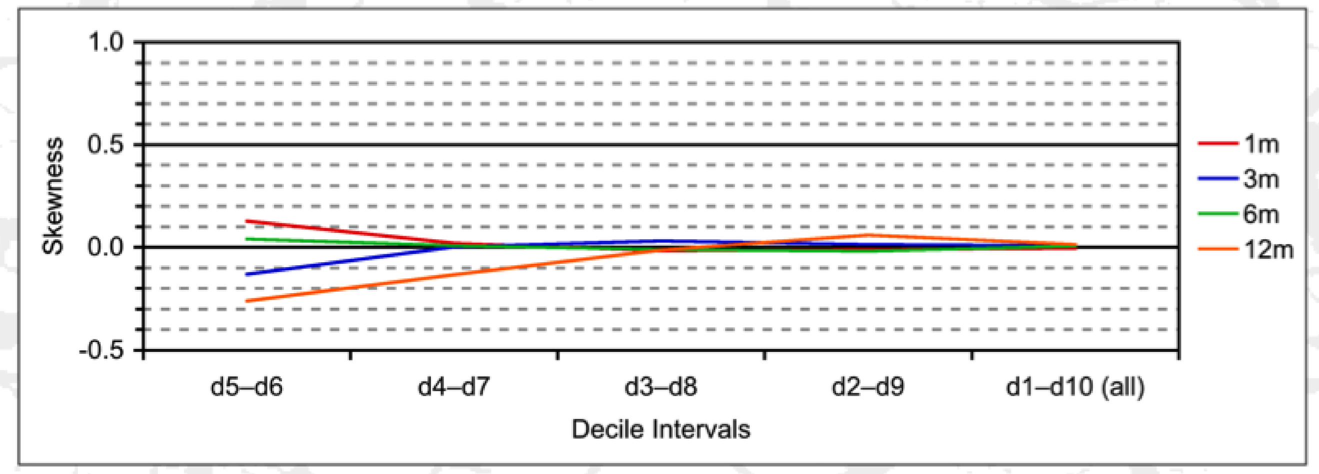

Fig 4a. Variation of skewness, root-normal SPIs.

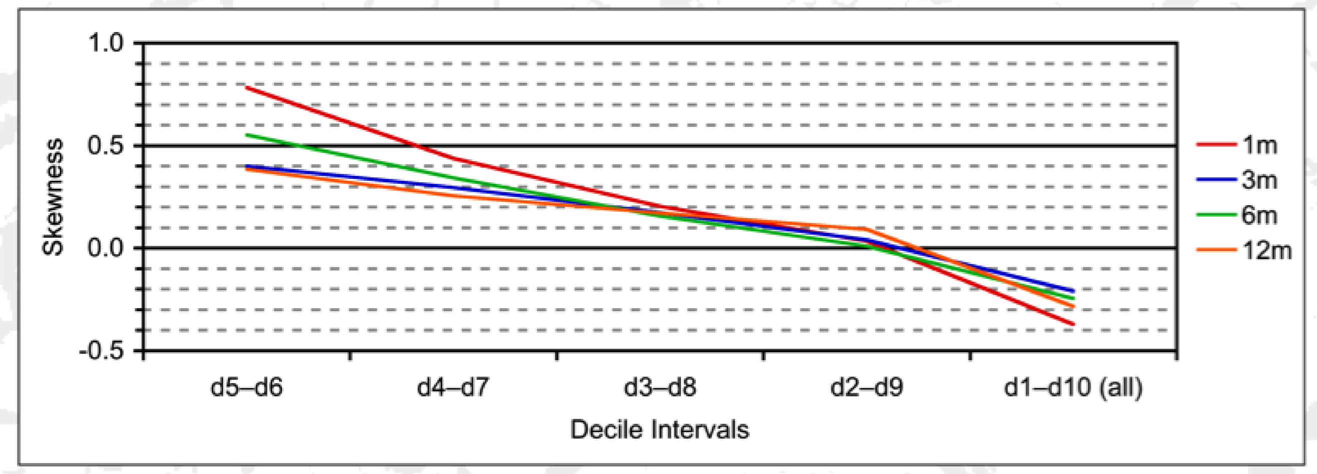

Fig 4b. Variation of skewness, McKee *et al.* SPIs.

Conclusions

Our investigation does not invalidate McKee *et al.*'s method: rather, it verifies the robustness of their basic premise in proposing SPIs as drought indices.

Whatever method is used to calculate SPIs, the accuracy of the SPI-set obtained is limited by the goodness-of-fit of the probability model used to characterise the precipitation data under investigation.

The advantages of direct, root-normal calculation are SPI-sets with means, st. deviations and skewnesses closer to the theoretical values (0, 1, 0 respectively) than equiprobability-mapped SPIs.

The disadvantages are a more complicated calculation which requires an iterative algorithm – can't be done on a spreadsheet, for example, unlike gamma-distribution fitting.

References

1. McKee T B, Doesken N J and Kleist J (1993). The relationship of drought frequency and duration to time scales. Preprints, 8^{th} Conference on Applied Climatology, pp. 179–184. January 17–22, Anaheim, California.
2. McKee T B, Doesken N J and Kleist J (1995). Drought monitoring with multiple time scales. Proceedings of the 9^{th} Conference on Applied Climatology, January 15-20, American Meteorological Society, Boston, Massachusetts. 233-236.
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