Monte-Carlo Simulation of Radon Equilibrium Under Varying Conditions.

R G M Crockett

Data Analysis Group, School of Science and Technology, University of Northampton, NN2 6JD, UK.

Introduction

At the core of this investigation is the development of Monte-Carlo simulations of the radon, 222Rn, and thoron, 220Rn, decay chains. Currently, these preliminary simulations assume an hypothetical closed cubic metre of atmosphere and the simulations are time-stepped at constant intervals. At each time-step, there is a radon (or thoron) influx and each nucleus in the decay chains decays probabilistically, with nuclei and activity being aggregated and tabulated at the end of each time-step.

The preliminary simulations have been coded in two open-source interpreted mathematical software packages:

- Scilab – user-friendly environment;
- Yorick – fast, good numerical precision.

The 222Rn Decay Chain

The full version of the 222Rn decay chain, including five very low probability decay paths, was modelled. This is shown in Figure 1.

\[ \begin{align*}
\alpha \quad & 222Rn \\
\beta & 218Po \\
\beta & 214Po \\
\beta & 214Bi \\
\beta & 214Pb
\end{align*} \]

Fig 1. The 222Rn Decay Chain (main decay shaded).

The Simulation – 222Rn

The simulation is a time-stepped Monte-Carlo simulation. Each daughter, from 222Rn (beginning) to 206Pb (end, stable) is represented by a loop which determines the random decay according to the probability of decay per time-step (calculated from tabulated half-life data).

Currently, the decays in the chain are processed sequentially. Thus, at each time-step, the daughters are ‘processed’ upstream, i.e. from 220Pb to 222Rn so that each daughter is ‘decayed’ according to the previous state before being updated.

Schematically, at each time-step, for each daughter in the decay chain, each daughter has also been investigated. This allows calculation of the radon equilibrium factor, \( F \).

\[ P(\text{decay}) = P(\text{daughter}, \text{timestep}, \text{decay}) \]

\[ \text{activity} = 0 \\
\text{nucleus} = \text{nucleus, old} \\
\text{for counter} = 1 \text{ to } \text{nucleus, old} \\
\text{if } P(\text{nucleus, decay}) \geq P(\text{decay}) \\
\text{nucleus} = \text{nucleus} - 1 \\
\text{nucleus, next} = \text{nucleus, next} + 1 \\
\text{end if} \\
\text{end for} \\
\text{nucleus, old} = \text{nucleus} \\
\text{plus downstream checks/updates} \]

The decay probabilities are listed in Table 1.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Decay Probability 5-minute timestep</th>
</tr>
</thead>
<tbody>
<tr>
<td>222Rnα</td>
<td>6.29 x 10^{-11}</td>
</tr>
<tr>
<td>220Rnβ</td>
<td>1.35 x 10^{-15}</td>
</tr>
<tr>
<td>218Poα</td>
<td>0.999</td>
</tr>
<tr>
<td>214Poα</td>
<td>0.001</td>
</tr>
<tr>
<td>214Bi</td>
<td>1.000</td>
</tr>
<tr>
<td>214Biβ</td>
<td>3.20 x 10^{-15}</td>
</tr>
<tr>
<td>214Pbβ</td>
<td>0.160</td>
</tr>
</tbody>
</table>

The ‘Experiments’.

1. Cyclically Varying Radon Influx

The main investigation so far has been into cyclically varying radon influx. The phase relationships between radon influx, radon concentration and the 222Rn, 218Po and 214Po α-particle activities have been investigated for 12h, 24h and 48h sinusoidal cycles in radon influx.

2. Radon Equilibrium Factor

The effect of different metallic-daughter plate-out probabilities on the equilibrium of 222Rn and its α-emitting daughters has also been investigated. This allows calculation of the radon equilibrium factor, \( F \).

\[ F = \frac{0.106n_{218Po} + 0.514n_{214Po} + 0.380n_{222Rn}}{n_{222Rn}} \]

Fig 2a. 12h Sinusoidal Variation in Radon Influx.

Fig 2b. 24h Sinusoidal Variation in Radon Influx.

Fig 3. Radon α-Particle Equilibrium.

Results: Cyclic Lags.

The cyclic relationships are shown in Figure 2, for 12h and 24h cycles. The lags for 12h, 24h and 48h cycles are shown in Table 2.

<table>
<thead>
<tr>
<th>Cycle Period</th>
<th>Lag of 222Rn concentration behind 222Rn influx (%) of period</th>
</tr>
</thead>
<tbody>
<tr>
<td>48h</td>
<td>31.31 (24.0%)</td>
</tr>
<tr>
<td>24h</td>
<td>51.51 (24.4%)</td>
</tr>
<tr>
<td>12h</td>
<td>56.56 (24.4%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cycle Period</th>
<th>Lag of 214Po α-activity behind 222Rn concentration (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48h</td>
<td>9 m 5 m</td>
</tr>
<tr>
<td>24h</td>
<td>7 m 7 m</td>
</tr>
<tr>
<td>12h</td>
<td>7 m 7 m</td>
</tr>
</tbody>
</table>

Tab 2. Lags: 48h, 24h and 12h Cycles (mean, 5 runs).

Note the ~1/4 cycle lag of 222Rn behind influx and the ~90m cycle lag of 214Po α-activity behind 222Rn.

Results: Radon Equilibrium Factor.

The radon equilibrium factor, \( F \), was calculated for plate-out rates as shown in Figure 3, according to the formula cited by the Wise Uranium Project.

Conclusion

These preliminary results, obtained using a coarse time-step, start to illuminate the behaviours of the radon-daughter system when the influx is cyclically varying: it is evident that the shorter the cyclic period, the more damped the response of the daughters to the variation.

However, it is clear that:

i) 222Rn concentration lags the influx by ~90m

ii) 214Po α-activity lags 222Rn and 218Po α-activity by ~90m

It is intended to recode the simulation to run on a multi-core HPC cluster such that daughters can be decayed in parallel rather than sequentially, to reduce the run-time and enable finer time-stepping.

The thoron, 220Rn, decay-chain will also be simulated: this will assist with understanding the equilibrium and cyclic behaviours in situations where both radon isotopes are present.

References

2. Scilab (http://www.scilab.org)

Fig 3a. Schematically, at each time-step, for each daughter in the decay chain.